Single-Passband Single-Stopband Narrow-Band Filters

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***Abstract*—Techniques are presented for obtaining an optimum filter response for narrow-band requirements of a single passband and a single stopband. For certain transformed equiripple passband responses, including Chebyshev and elliptic function responses and a new "double $n/2$ poles" response, the minimum number of resonators can be easily determined, resulting in reduced size and weight relative to more conventional frequency-symmetric bandpass or bandstop filters.**

***Index Terms*—Bandpass filters, Chebyshev filters, circuit synthesis, elliptic filters, passive filters, resonator filters.**

I. Introduction

A NARROW bandpass filter specification occasionally appears that requires only a single, relatively narrow stopband. The most efficient filter response (lowest value of n, the number of resonators) for this requirement will be an asymmetric response with all n reflection zeros in the specified passband and all n loss poles in the specified stopband. Conventional filters such as a bandpass filter with lower and upper stopbands, or a bandstop filter with lower and upper passbands, will necessarily have a less efficient response (more resonators) in order to meet this requirement. Rhodes [1] presented techniques to obtain single-stopband responses and low-pass or high-pass types of passbands, based on Chebyshev and elliptic-function prototypes.

In this paper, stopbands of frequency-symmetric narrow bandpass filters will be transfomed to contiguous bands below or above the passband using the narrow-bandpass transformed frequency variable. Transformations will be demonstrated for the Chebyshev response, the elliptic function response (the optimum response when an equiripple passband and a constant level of minimum stopband loss are specified), and a response of intermediate selectivity referred to as "double $n/2$ poles."

II. Definitions

A. Transformed Variable

For a narrow bandpass filter with an equiripple passband from f_1 to f_2, it is convenient to perform synthesis (approximation of a specification and prototype realization) in the transformed frequency variable z defined by [2]

$$z = \sqrt{\frac{f - f_2}{f - f_1}}, \qquad \mathrm{Re}(z) \geq 0. \tag{1}$$

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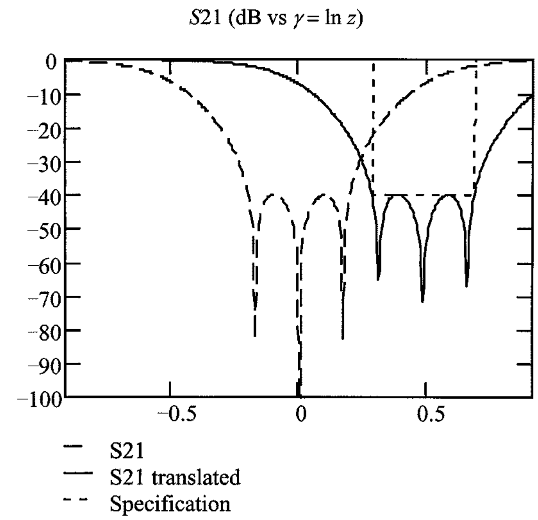

Fig. 1. Translation of $n = 3$ elliptic function response in $\gamma = \ln z$.

The passband maps onto $z^2 \leq 0$, and the stopband maps into $0 < z^2 < \infty$. A further transformation of $\gamma = \ln z$ maps the stopband into $-\infty < \gamma < \infty$.

In the normalized low-pass frequency domain, with passband corners at $\omega = \pm 1$, transformation (1) and its inverse are

$$z = \sqrt{\frac{\omega - 1}{\omega + 1}}, \qquad \mathrm{Re}(z) \geq 0 \tag{2a}$$

$$\omega = \frac{1 + z^2}{1 - z^2}. \tag{2b}$$

The low-pass prototype circuit response functions [2] are rational functions in ω and z^2, and therefore a bilinear transformation in ω or z^2 will scale the frequencies but otherwise preserve the filter response. Furthermore, the transformation in z that maintains the passband edges ($z \to \infty$ and $z = 0$) is the simple linear transformation $z \to az$ where a is a positive, real constant. This transformation preserves the passband corners f_1 and f_2 and may be used to map the stopband into a contiguous segment entirely below or above the passband.

In the transformed variable $\gamma = \ln z$, the stopband transformation is a simple translation where $\gamma \to \gamma + \ln a$. Fig. 1 shows the translation of the stopband loss poles of a degree three elliptic function filter from a set which is symmetric about $\gamma = 0$ ($z^2 = 1$ or $\omega \to \infty$, i.e., a symmetric frequency response), to a stopband for which $\gamma > 0$, clearly retaining the equal-minima characteristics. The result is a stopband that has been shifted entirely to the lower side of the passband ($-\infty < \omega < -1$), as will be shown in Section III.

B. Degree Equations

Chebyshev and elliptic function filters are specific types of general Chebyshev rational function filters, all of which exhibit equal ripple passband responses, regardless of the distribution of their loss poles. Another type, the "double $n/2$ poles" filter, will also be described.

The "degree equation" of a symmetric-response low-pass filter relates the degree n and the filter selectivity to the minimum passband return loss ripple RL and the minimum stopband loss SL. The selectivity is represented by the stopband corners at $\omega = \pm\omega_s$ in the normalized low-pass frequency domain. As presented here, the degree equations are not exact, but have negligible error for values of RL and SL which are greater than 15 dB each.

1) Chebyshev Filter: For a low-pass Chebyshev filter, all n loss poles are at infinite frequency ($z^2 = 1$). From any of several formulas for the stopband loss of a Chebyshev filter, the degree equation is

$$n \cdot 20 \log\left(\omega_s + \sqrt{\omega_s^2 - 1}\right) \geq RL + SL + 6.02 \tag{3}$$

with RL and SL in decibels.

2) Double $n/2$ Poles Filter: This filter is of even degree with two loss poles, each of order $n/2$, at $\pm\omega_p$ and a minimum stopband loss SL at infinite frequency. The loss poles and stopband corners are given by

$$\omega_p = \frac{1}{2}\left(\sqrt{g_n} + \frac{1}{\sqrt{g_n}}\right)$$
$$\omega_s = \frac{1}{2}\left(\sqrt{\frac{g_n + 1/g_n}{2}} + \frac{1}{\sqrt{\frac{g_n + 1/g_n}{2}}}\right)$$
$$g_n \equiv [4(10^{RL/10} - 1)(10^{SL/10} - 1)]^{1/n}.$$

The double $n/2$ poles filter may be useful for applications where the multiple-ordered loss poles will mitigate the reduction in skirt selectivity due to dissipation in low-Q resonators. The degree equation is

$$n \cdot 10 \log\left(\frac{1+u}{1-u}\right) \geq RL + SL + 6.02$$
$$u \equiv (1 - 1/\omega_s^2)^{1/4}. \tag{4}$$

3) Elliptic Function Filter: Darlington's degree equation [3] for a low-pass elliptic function filter, in which the n loss poles are distributed to obtain an optimum equal-minima stopband response, is [4], [5]

$$n \cdot 10 \log\left[\frac{1}{q(k)}\right] \geq RL + SL + 12.04, \qquad k = 1/\omega_s. \tag{5}$$

Accurate methods of calculating the elliptic modulus k from the nome q and q from k are given by Orchard [5] and are not repeated here.

III. Stopband Transformations

A. Minimum Degree

For a specified passband from p_1 to p_2 and a specified stopband from s_1 to s_2, the minimum number of resonators required for a transformed response can be calculated from (3)–(5). Using the specified passband corner frequencies as the equiripple passband edges (i.e., $f_1 = p_1$ and $f_2 = p_2$), first map the stopband corners s_1 and s_2 into the z-plane stopband corners z_1 and z_2 using (1).

The scale factor that will remap z_1 and z_2 into a frequency-symmetric response is

$$a = \frac{1}{\sqrt{z_1 z_2}}.$$

The corners z_1 and z_2 map into corresponding stopband corners $\pm\omega_s$ in the normalized low-pass frequency domain, where

$$\omega_s = \frac{z_2 + z_1}{z_2 - z_1}.$$

This value of ω_s is then used to calculate the minimum number of resonators.

B. Maximum Bandwidth

Having chosen any value of n that is greater than that given by (3)–(5), a response may be optimized by maximizing the passband width for a specified stopband. Because of the one-sided stopband, the highest values of passband delay and incidental dissipation resulting from finite resonator Q will occur at p_1 for a lower stopband or at p_2 for an upper stopband. To minimize these distortions, the equiripple passband edge opposite the stopband is fixed at the corresponding specified passband corner, and the equiripple passband edge adjacent to the stopband is adjusted to meet the stopband specification with no frequency margin. This is accomplished by reversing the transformations used to determine the minimum n.

Let z_1 and $z_2 = 1/z_1$ now refer to the z-plane stopband corners in the initial frequency-symmetric low-pass prototype response, which are transforms of $\omega = \pm\omega_s$. These z-plane stopband corners will be scaled to correspond to the desired stopband corners s_1 and s_2 using the linear (in z^2) transformation

$$z^2 \rightarrow \frac{z^2}{z_1^2}\,\frac{s_1 - f_2}{s_1 - f_1} = \frac{z^2}{z_2^2}\,\frac{s_2 - f_2}{s_2 - f_1}. \tag{6}$$

Fixing one of the equiripple passband edges as described above, the other edge may be found by solving (6), resulting in

Lower Stopband

$$f_1 = \frac{s_1 \cdot z_1^2(s_2 - p_2) - s_2 \cdot z_2^2(s_1 - p_2)}{z_1^2(s_2 - p_2) - z_2^2(s_1 - p_2)}$$
$$f_2 = p_2$$

Upper Stopband

$$f_1 = p_1$$
$$f_2 = \frac{s_1 \cdot z_2^2(s_2 - p_1) - s_2 \cdot z_1^2(s_1 - p_1)}{z_2^2(s_2 - p_1) - z_1^2(s_1 - p_1)}.$$

For prototype realization [2], the initial (frequency-symmetric) stopband loss poles will be similarly scaled using (6).

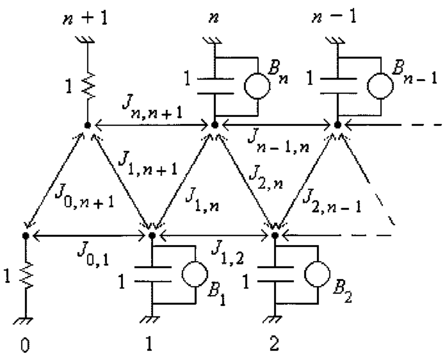

Fig. 2. Canonical asymmetric coupled resonator prototype.

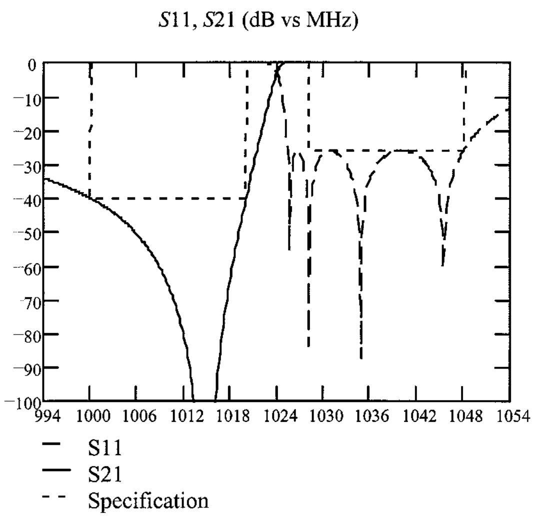

Fig. 3. Transformed $n = 4$ Chebyshev response.

C. Examples

Consider a requirement to stop from $s_1 = 1000$ to $s_2 = 1020$ MHz, with $SL = 40$ dB minimum loss, and to pass from $p_1 = 1028$ to $p_2 = 1048$ MHz, with $RL = 26$ dB minimum return loss. Then $z_1 = 1.309307$, $z_2 = 1.870829$, and $\omega_s = 5.663429$.

A canonical asymmetric coupled-resonator prototype [6] of the form shown in Fig. 2, where $B_i \equiv J_{i,\,i}$ are the on-diagonal elements of the coupling matrix J, will be realized for each of the transformed frequency-asymmetric responses.

1) Chebyshev Filter: From (3), $n \geq 3.427291 \rightarrow 4$. For $n = 4$, the calculated lower passband equiripple edge is $f_1 = 1025.38$ MHz. The resulting lossless filter response is shown in Fig. 3, and the prototype coupling matrix is as shown in (7) at the top of the following page.

2) Double $n/2$ Poles Filter: From (4), $n \geq 2.997846 \rightarrow 4$, since the degree of this filter must be even. For $n = 4$, $f_1 = 1023.21$ MHz, and the resulting lossless filter response is shown in Fig. 4; the prototype coupling matrix is as shown in (8) at the top of the following page.

3) Elliptic Function Filter: The nome is calculated to be $q = 0.001979605$, and from (5), $n = 2.886712 \rightarrow 3$. For $n = 3$, the calculated lower passband equiripple edge is $f_1 = 1027.07$ MHz, resulting in the lossless filter response shown in Fig. 5, realized by the prototype coupling matrix in the matrix as shown in (9) at the top of the following page.

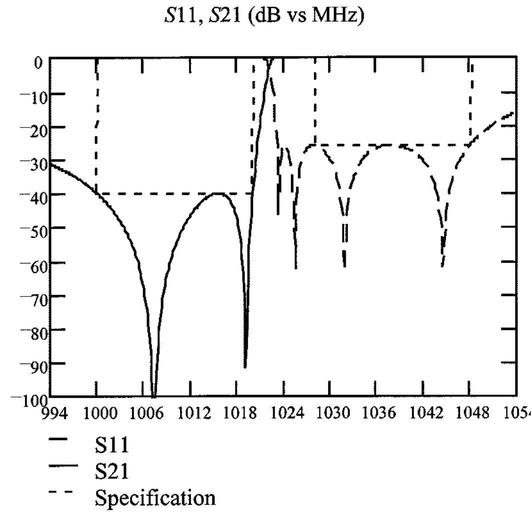

Fig. 4. Transformed $n = 4$ double $n/2$ poles response.

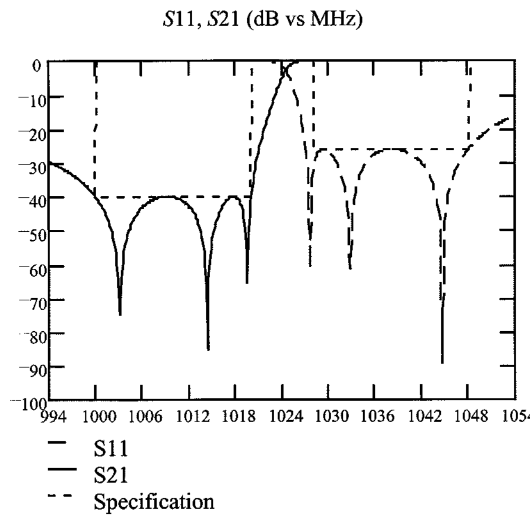

Fig. 5. Transformed $n = 3$ elliptic function response.

IV. Conclusions

For the special filtering requirements of a single passband and a single stopband, a technique has been demonstrated for determining the minimum number of resonators required and maximizing the equiripple bandwidth to achieve a minimum distortion. This technique was demonstrated specifically for three types of Chebyshev rational function responses.

By applying this technique to the elliptic function filter, one may determine the absolute minimum number of resonators required to meet a single-passband single-stopband specification requiring a constant minimum stopband loss.

Because of the asymmetric frequency response, a resulting coupled-resonator prototype network may be more difficult to implement than a conventional frequency-symmetric bandpass or bandstop prototype, even though it requires fewer resonators.

$$J = \begin{bmatrix} 0 & 1.347654 & 0 & 0 & 0 & 0.117872 \\ 1.347654 & -0.625757 & 1.02198 & 0 & 0.900809 & -0.563946 \\ 0 & 1.02198 & 0.668265 & 0.072487 & -0.580546 & 0 \\ 0 & 0 & 0.072487 & 1.048528 & 0.400851 & 0 \\ 0 & 0.900809 & -0.580546 & 0.400851 & 0.204333 & 1.223983 \\ 0.117872 & -0.563946 & 0 & 0 & 1.223983 & 0 \end{bmatrix} \quad (7)$$

$$J = \begin{bmatrix} 0 & 1.425723 & 0 & 0 & 0 & 0.208023 \\ 1.425723 & -0.618009 & 0.867467 & 0 & 0.881508 & -0.783768 \\ 0 & 0.867467 & 0.875759 & 0.023997 & -0.431959 & 0 \\ 0 & 0 & 0.023997 & 1.054242 & 0.269665 & 0 \\ 0 & 0.881508 & -0.431959 & 0.269665 & 0.542226 & 1.190963 \\ 0.208023 & -0.783768 & 0 & 0 & 1.190963 & 0 \end{bmatrix} \quad (8)$$

$$J = \begin{bmatrix} 0 & 1.484015 & 0 & 0 & -0.224269 \\ 1.484015 & -0.591074 & 0.767753 & -0.921961 & 0.890584 \\ 0 & 0.767753 & 1.138276 & 0.383806 & 0 \\ 0 & -0.921961 & 0.383806 & 0.792291 & 1.187081 \\ -0.224269 & 0.890584 & 0 & 1.187081 & 0 \end{bmatrix} \quad (9)$$

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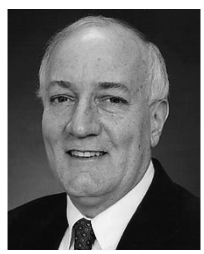

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