Efficient Sensitivity Analysis of Transmission-Line Networks Using Model-Reduction Techniques

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***Abstract*—An efficient algorithm, based on congruent transformation and model reduction, is proposed for evaluation of frequency- and time-domain sensitivity of large linear networks containing lossy coupled transmission lines. The sensitivity of the voltage and current waveforms can be calculated with respect to lumped components and parameters of transmission lines. The algorithm is based on projecting the adjoint network equations on a reduced-order subspace that preserves the circuit moments. The proposed algorithm provides a significant decrease in the computational expense for sensitivity analysis.**

***Index Terms*—Circuit simulation, multiconductor transmission lines, reduced-order systems, sensitivity.**

I. Introduction

ADVANCES IN fabrication methods and decreasing device sizes have significantly increased the relative contribution of high-speed interconnects to overall signal degradation [1]. At higher frequencies, interconnects are modeled by transmission lines. Transmission-line analysis has, therefore, become imperative at all levels of the design hierarchy from backplanes to printed circuit boards, packages, and integrated circuits [2]. The analysis of large systems containing transmission lines have presented additional problems to circuit simulation, stemming from the fact that such distributed elements are modeled by partial differential equations as opposed to ordinary differential equations, as in the case of lumped elements. A significant amount of research has been done to overcome these difficulties and reduce the CPU cost of transmission-line analysis [3].

Recently, techniques based on congruent transformation have emerged as an efficient tool to generate reduced-order passive models of very large linear networks. These techniques have been applied successfully to lumped [4] and distributed networks [5], [6] resulting in significant savings in CPU time. Such methods allow the circuit designer to determine the response of a given circuit.

Several techniques can be found in the literature that use model reduction based on Padé approximation to perform sensitivity analysis of linear *lumped* networks [7], [8]. Besides the fact that these techniques cannot handle distributed networks, their analysis requires finding the derivatives of the poles and zeros of the reduced model. In this paper, we propose an efficient algorithm for sensitivity analysis of transmission-line networks based on congruent transformation techniques. The proposed technique handles distributed transmission-line circuit stamps including frequency-dependent parameters without the need for discretization of the telegrapher's equations. The algorithm is based on projecting the adjoint network equations on a reduced-order subspace that preserves the circuit moments. The sensitivity can then be evaluated over any number of frequency points for the cost of one QR factorization. The remainder of the paper is organized as follows. Section II presents the formulation of the circuit equations. This is followed in Section III by a brief description of adjoint sensitivity analysis. The proposed model-reduction algorithm is then discussed in Section IV. Finally, the results and conclusion are presented in Sections V and VI, respectively.

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II. Formulation of Network Equations

Consider a linear network ϕ containing linear lumped components and N_t lossy coupled transmission-line sets, with n_k coupled conductors in transmission-line set k. Assume the network ϕ has N nodal variables, the modified nodal analysis (MNA) matrix equations for the network with impulse input excitation can be formulated as

$$\mathbf{A}(s)\mathbf{X}(s) = \mathbf{B} \tag{1}$$

$$\mathbf{A}(s) = \mathbf{H} + s\mathbf{W} + \mathbf{Y}(s) \tag{2}$$

where

- $\mathbf{X}(s) \in C^N$ vector of the N complex variables describing the subnetwork ϕ, which includes node voltage waveforms appended by independent and dependent voltage source currents and inductor currents;
- $\mathbf{W}, \mathbf{H} \in \Re^{N\times N}$ constant matrices describing the lumped memory and memoryless elements of network ϕ, respectively. $\mathbf{W}, \mathbf{H}$ are formulated as suggested in [4];
- $\mathbf{B} \in C^N$ vector with entries determined by the independent voltage and current sources;
- $\mathbf{Y}(s)$ $= \sum_{k=1}^{N_t} \mathbf{D_k}\mathbf{Y_k}(s)\mathbf{D_k^t}$ is a matrix containing the frequency dependent y-parameters of all the distributed transmission-line sets.

$\mathbf{D_k} = [d_{i,j}]$ with elements $d_{i,j} \in \{0,1\}$, where $i \in \{1,2,\ldots,N\}$, $j \in \{1,2,\ldots,2n_k\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $\mathbf{I}_k(t) \in \Re^{2n_k}$, the vector of currents entering the interconnect subnetwork k, into the node space $\Re^N$ of the network. The y-parameters of transmission-line set k are given by [3]

$$\mathbf{Y}_k\mathbf{V}_k = \mathbf{I}_k \tag{3}$$

where $\mathbf{V}_k(t) \in \Re^{2n_k}$ is the vector of terminal voltages of the interconnect subnetwork k and

$$\mathbf{Y}_k = \begin{bmatrix} \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} \\ \mathbf{S}_i\mathbf{E}_2\mathbf{S}_v^{-1} & \mathbf{S}_i\mathbf{E}_1\mathbf{S}_v^{-1} \end{bmatrix}. \tag{4}$$

$\mathbf{E}_1$ and $\mathbf{E}_2$ are the diagonal matrices

$$\mathbf{E}_1 = \mathrm{diag}\left\{\frac{1+e^{-2\gamma_i d}}{1-e^{-2\gamma_i d}},\ i=1\ldots N_k\right\} \tag{5}$$

$$\mathbf{E}_2 = \mathrm{diag}\left\{\frac{2}{e^{-\gamma_i d}-e^{\gamma_i d}},\ i=1\ldots N_k\right\} \tag{6}$$

where d is the length of the transmission-line set, and γ_i^2 are the eigenvalues of the matrix $\mathbf{Z}_L\mathbf{Y}_L$ with associated eigenvectors $\mathbf{x}_i$, where

$$\mathbf{Z}_L = \mathbf{R} + s\mathbf{L} \tag{7}$$

$$\mathbf{Y}_L = \mathbf{G} + s\mathbf{C}. \tag{8}$$

$\mathbf{R}$, $\mathbf{L}$,$\mathbf{C}$, and $\mathbf{G} \in \Re^{N_k\times N_k}$ are the per-unit-length resistance, inductance, capacitance, and conductance matrices of the transmission-line set k. $\mathbf{S}_v$ is a matrix with the eigenvectors $\mathbf{x}_i$ in the columns, and $\mathbf{S}_i = \mathbf{Z}_L^{-1}\mathbf{S}_v\Gamma$, where $\Gamma = \mathrm{diag}\{\gamma_i, i = 1\ldots N_k\}$. It is to be noted that the per-unit-length $\mathbf{R}$, $\mathbf{L}$, $\mathbf{C}$, $\mathbf{G}$ parameters of the transmission line can also be a function of frequency.

III. Sensitivity Analysis

Several techniques can be found in the literature to determine the sensitivity of the output with respect to any parameter λ in the network. Amongst these, the adjoint technique has been proven to be the most efficient method [11]. This section briefly describes this technique. This forms a basis to propose the new algorithm discussed in Section IV.

With λ being a circuit parameter representing a lumped or transmission-line element, (1) can be rewritten as

$$\mathbf{A}(s,\lambda)\mathbf{X}(s,\lambda) = \mathbf{B}. \tag{9}$$

Let V be the output variable of interest. We have

$$V = \mathbf{d}^t\mathbf{X} \tag{10}$$

where $\mathbf{d}$ is a selector vector with nonzero entries corresponding to the output variable. The sensitivity of V with respect to λ can be written as

$$\frac{\partial V}{\partial \lambda} = -\mathbf{X}_a^t\frac{\partial \mathbf{A}}{\partial \lambda}\mathbf{X} \tag{11}$$

where $\mathbf{X}$ is the solution of the network equations, and $\mathbf{X}_a$ is the solution of the adjoint equations

$$\mathbf{A}^t(s,\lambda)\mathbf{X}_a = \mathbf{d}. \tag{12}$$

Equations (9), (11), and (12) can be solved to obtain the sensitivity of V with respect to λ. The relative sensitivity is defined as

$$S_\lambda = \lambda\frac{\partial V}{\partial \lambda}. \tag{13}$$

The definition in (13) is directly related to the partial derivatives, however it is more meaningful when comparing sensitivity with respect to components of different magnitudes.

The computational cost of this method is one L/U factorization of $\mathbf{A}(s,\lambda)$ at each frequency point. It is to be noted that, for large circuits, this method can be computationally intensive. To address this problem, in Section IV, we propose an algorithm to perform sensitivity analysis on large networks using model-reduction techniques.

IV. Sensitivity Analysis Using Model Reduction

The proposed algorithm to find the sensitivity of voltage and current waveforms of the transmission-line network described in (1) is presented in the following subsections. First, an algorithm is presented for the reduction of the equations representing the original system (1) and its sensitivity (12). This is followed by a description of the computation of the sensitivity of the MNA matrix, required in (11). A pseudocode of the algorithm is given in Fig. 1.

A. Model Reduction

The original system in (1) can be reduced by a congruent transformation into a smaller system of size q [6]. The congruent transformation matrix $\mathbf{Q}$ is an orthonormal basis, constructed such that

$$\mathrm{span}(\mathbf{Q}) = \mathrm{span}[\mathbf{M}_0 \quad \cdots \quad \mathbf{M}_q] \tag{14}$$

where $\mathbf{Q} \in \Re^{N\times q}$. M_i is the ith moment of the circuit response. The moments M_i are evaluated using the recursive relation given by [10]

$$[\mathbf{A}]|_{s=0}\mathbf{M}_i = -\sum_{r=1}^{n}\frac{\left.\dfrac{\partial^r\mathbf{A}}{\partial s^r}\right|_{s=0}\mathbf{M}_{i-r}}{r!} \tag{15}$$

with

$$[\mathbf{A}]|_{s=0}\mathbf{M_0} = \mathbf{B}. \tag{16}$$

The reduced system is formed by performing the congruent transformation

$$\mathbf{X}(s) = \mathbf{Q}\hat{\mathbf{X}}(s) \tag{17}$$

on the original system (1). Using (17) and (1), the reduced system can be written as

$$\hat{\mathbf{A}}(s)\hat{\mathbf{X}}(s) = \hat{\mathbf{B}} \tag{18}$$

where

$$\hat{\mathbf{A}} = \mathbf{Q}^t\mathbf{A}\mathbf{Q} \quad \hat{\mathbf{B}} = \mathbf{Q}^t\mathbf{B}. \tag{19}$$

```
[H,W,Y(s),B,d] = FormMna();
A(s) =H+W+Y(s);

// Form reduced order model
[K,K_a]= getMoments(A ,B,d);
Q = orthonormalize(K);
Q_a = orthonormalize(K_a);
Â(s)  = Q'A(s)Q;
Â_a(s)  = Q_a'A'(s)Q_a;
B̂  = Q'B;
d̂_a  = Q_a'd;

// Perform sensitivity analysis on reduced system
for each frequency point
   solve Â(s)x̂ = B̂ ;    solve Â_a(s)x̂_a = d_a
         x = Qx̂  ;             x_a = Q_a x̂_a
   evaluate ∂A/∂λ as described in secion IV.B

   ∂V/∂λ = x_a^t(∂A/∂λ)x
endfor
quit;

function orthonormalize(A)
[m,n] = size(A)
for i = 1 to min(m-1,n)
   u_i = A(i:m,i) + sign(A(i,i))e_1  // e_1 is the first column of the
   u_i = u_i/norm(u_i)               // identity matrix
   A(i:m,i:n) = A(i:m,i:n) - 2u_i(u_i^T A(i:m,i:n))
end for
return(A);
end function

function getMoments(A ,B,d)
   [L,U] = lu(A );
   sum = 0; sum2 = 0;
   for i = 1 to num_moments_needed
   for k = 1 to i-1
      sum -= kth_derivative_of_A*N(n-k)/factorial(i);
      sum2 -= transpose(kth_derivative_of_A)*M(n-k)/factorial(i);
      /* The kth derivative of A(s) is computed as described
         in Section IV.A.*/
   end for
   N_i = forward_backward_subst(L,U,sum);
   M_i = forward_backward_subst(U',L',sum2);
   end for
   return [N,M]
end function
```

Fig. 1. Pseudocode of the algorithm.

The solution of the original network ϕ is obtained by mapping the solution of the reduced circuit back to the original space using (17). Such a congruent transformation was shown to produce a passive system while preserving the first q derivatives of the original system [6].

From (17), it can be seen that

$$\frac{\partial \mathbf{X}}{\partial \lambda} = \frac{\partial \mathbf{Q}}{\partial \lambda}\hat{\mathbf{X}} + \mathbf{Q}\frac{\partial}{\partial \lambda}\hat{\mathbf{X}}. \tag{20}$$

Equation (20) demonstrates that finding the sensitivity of $\mathbf{X}$ with respect to λ involves finding the derivative of the congruent transformation $\mathbf{Q}$ with respect to λ, which is difficult and cumbersome to evaluate. In order to avoid the evaluation of $\partial\mathbf{Q}/\partial\lambda$, we perform model reduction directly on the adjoint system of (12) through a congruent transformation $\mathbf{Q}_a$. As in the case of model reduction of the original system ϕ, the moments of $\mathbf{X}_a$ are needed for the construction of the congruent transformation $\mathbf{Q}_a$. These moments are evaluated using the recursive relationship given by [10]

$$\left[\mathbf{A}^t\right]|_{s=0}\mathbf{M}_i^a = -\sum_{r=1}^{i}\frac{\left.\frac{\partial^r \mathbf{A}^t}{\partial s^r}\right|_{s=0}\mathbf{M}_{i-r}^a}{r!} \tag{21}$$

with

$$\left[\mathbf{A}^t\right]|_{s=0}\mathbf{M}_\mathbf{0}^a = \mathbf{d} \tag{22}$$

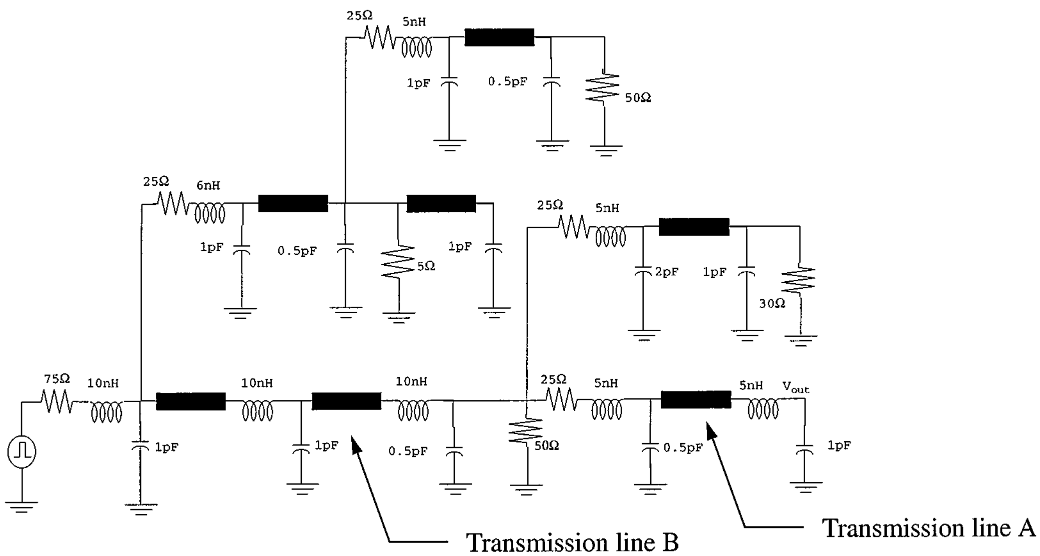

Fig. 2. Circuit for example 1.

where $\mathbf{M}_i^a$ is the ith moment of the adjoint circuit response. It should be noted that the factorization of $\mathbf{A}^t$ can be easily deduced from that of $\mathbf{A}$, which was evaluated in (16). Reducing the adjoint circuit equation (12), we have

$$\hat{\mathbf{A}}_a(s)\hat{\mathbf{X}}_a(s) = \hat{\mathbf{d}}_a \tag{23}$$

where

$$\hat{\mathbf{A}}_a = \mathbf{Q}_a^t \mathbf{A}^t \mathbf{Q}_a \quad \hat{\mathbf{d}}_a = \mathbf{Q}_a^t \mathbf{d}. \tag{24}$$

The solution of the adjoint network can, therefore, be found by mapping $\hat{\mathbf{X}}_a$ back to the original space through the congruent transformation

$$\mathbf{X}_a(s) = \mathbf{Q}_a \hat{\mathbf{X}}_a(s). \tag{25}$$

The sensitivity can then be found at any frequency by simply solving the reduced systems in (18) and (23). Such congruent transformation applied to a linear system, is shown to preserve the first q derivatives with respect to frequency, thus preserving the accuracy of the response [6]. In addition, the matrices involved in (18) and (23) are very small when compared to the original equations (9) and (12). Hence, the solution of the reduced system of equations is much less computationally expensive when compared to that of the original system. It is to be noted that the system moments and the adjoint moments are computed using the same L/U decomposition in (15) and (21). However, there is no obvious relationship between $\mathbf{Q}$ and $\mathbf{Q}_a$. Therefore two separate QR decompositions are required to find the two transformation matrices. Furthermore, the algorithm does not require $\mathbf{Q}$ and $\mathbf{Q}_a$ to be of the same order, although using the same order for both transformations is recommended.

B. Calculation of $\partial \mathbf{A}/\partial \lambda$

From (11), it can be seen that the derivative of the MNA equations is required to calculate the output sensitivities. The method of calculating this derivative, however, depends on the type of the parameter λ. The following subsections briefly describe the computation of $\partial \mathbf{A}/\partial \lambda$ when λ represents lumped and distributed elements.

1) Sensitivity with Respect to Parameters of Lumped Components: When λ represents a parameter of a lumped element, it can be seen from (2) and (11) that

$$\frac{\partial \mathbf{A}}{\partial \lambda} = \frac{\partial \mathbf{H}}{\partial \lambda} + s\frac{\partial \mathbf{W}}{\partial \lambda} \tag{26}$$

which can be easily evaluated [11].

2) Sensitivity with Respect to Electrical Parameters of Transmission Lines: If λ represents an electrical parameter of a distributed element, it can be seen from (2), (3), and (11) that

$$\frac{\partial \mathbf{A}}{\partial \lambda} = \frac{\partial}{\partial \lambda}\mathbf{Y}(s) = \frac{\partial}{\partial \lambda}\sum_{k=1}^{N_t} \mathbf{D_k Y_k}(s)\mathbf{D}_\mathbf{k}^t. \tag{27}$$

In order to compute $\partial \mathbf{A}/\partial \lambda$ using (27), we need to find the derivatives of $\mathbf{Y_k}(s)$ with respect to λ. It can be shown that $\mathbf{Y_k}(s)$ satisfies the relation [9]

$$\begin{aligned}\frac{\partial \mathbf{Y}_k}{\partial \lambda}\begin{bmatrix} S_v & 0 \\ 0 & S_v\end{bmatrix} &= \begin{bmatrix} \frac{\partial S_i}{\partial \lambda} & 0 \\ 0 & \frac{\partial S_i}{\partial \lambda}\end{bmatrix}\begin{bmatrix} E_1 & E_2 \\ E_2 & E_1\end{bmatrix} \\ &\quad + \begin{bmatrix} S_i & 0 \\ 0 & S_i\end{bmatrix}\begin{bmatrix} \frac{\partial E_1}{\partial \lambda} & \frac{\partial E_2}{\partial \lambda} \\ \frac{\partial E_2}{\partial \lambda} & \frac{\partial E_1}{\partial \lambda}\end{bmatrix} \\ &\quad - \mathbf{Y}_k \begin{bmatrix} \frac{\partial S_v}{\partial \lambda} & 0 \\ 0 & \frac{\partial S_v}{\partial \lambda}\end{bmatrix}.\end{aligned} \tag{28}$$

From (28), it can be seen that the sensitivity of $\mathbf{Y}_k$ depends on the partial derivatives of the eigenvalues γ_i^2 and eigenvectors $\mathbf{x}_i$ of the matrix $\mathbf{Z}_L\mathbf{Y}_L$, which can be obtained using

$$\begin{bmatrix} \gamma_i \mathbf{U} - \mathbf{Z}_L\mathbf{Y}_L & x_i \\ \mathbf{x}_i^t & 0\end{bmatrix}\begin{bmatrix} \frac{\partial \mathbf{x}_i}{\partial \lambda} \\ \frac{\partial \gamma^2}{\partial \lambda} i\end{bmatrix} = \begin{bmatrix} \left(\frac{\partial}{\partial \lambda}(Z_L Y_L)\right)\mathbf{x}_i \\ 0\end{bmatrix} \tag{29}$$

where $\mathbf{U}$ is the identity matrix.

3) Sensitivity with Respect to Physical Parameters of Transmission Lines: While sensitivity with respect to electrical pa-

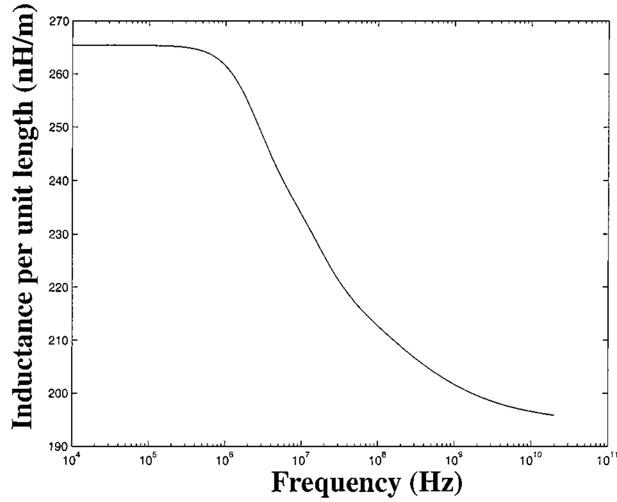

Fig. 3. Inductance per unit length of transmission-line B.

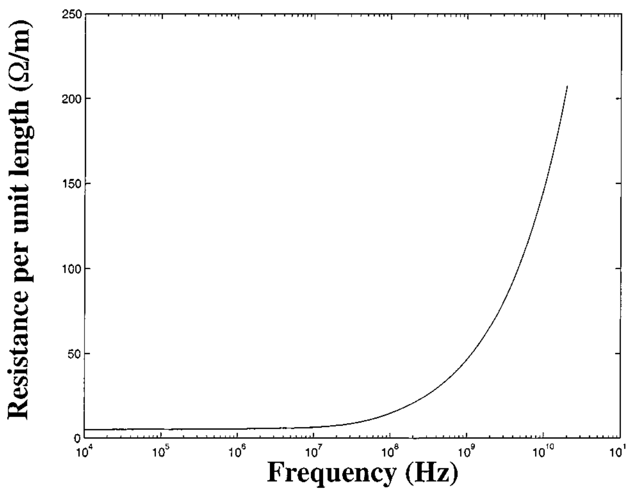

Fig. 4. Resistance per unit length of transmission-line B.

rameters of transmission lines is a useful measure, it is often an intermediate step in the calculation of sensitivity with respect to physical parameters such as width because ultimately physical parameters are the design parameters. Suppose λ is a physical parameter of a transmission-line system with N conductors. A simple expression for the output sensitivity can be written as

$$\frac{\partial V}{\partial \lambda} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\partial V}{\partial R_{i,j}}\frac{\partial R_{i,j}}{\partial \lambda} + \frac{\partial V}{\partial L_{i,j}}\frac{\partial L_{i,j}}{\partial \lambda} + \frac{\partial V}{\partial G_{i,j}}\frac{\partial G_{i,j}}{\partial \lambda} + \frac{\partial V}{\partial C_{i,j}}\frac{\partial C_{i,j}}{\partial \lambda}\right). \quad (30)$$

The above formulation requires the computation of the sensitivity of the output with respect to all $4 \times N^2$ electrical parameters of the transmission lines. For example, an interconnect containing four coupled lines will have 65 electrical parameters (including the length), but only ten physical parameters. Since the number of physical parameters is much smaller than that of the electrical parameters it is much more efficient to include the physical parameters directly in (29). The derivatives of the electrical parameters with respect to the physical parameters can be found using numerical methods [13]–[15], but for many cases, simple analytical expressions for the electrical parameters (such as those in [16]) can be used.

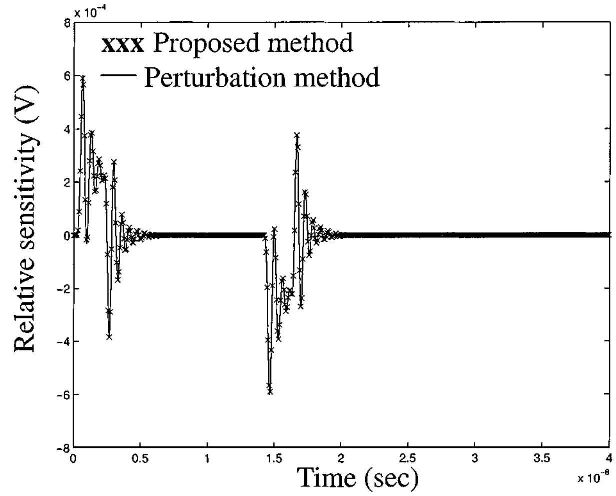

Fig. 5. Relative sensitivity of $V_{\rm out}$ with respect to capacitance per unit length of transmission line A.

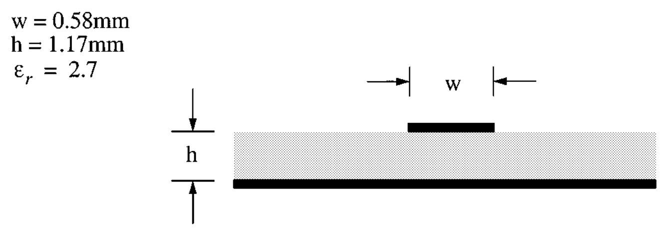

Fig. 6. Microstrip line from the numerical example.

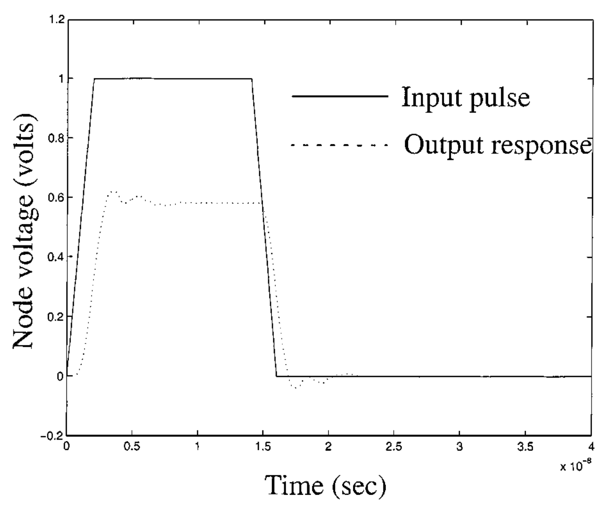

Fig. 7. Time-domain response.

V. Numerical Example

Example 1: To demonstrate the accuracy of the proposed algorithm, a circuit (Fig. 2) containing a transmission line with frequency-dependent parameters was considered. Figs. 3 and 4 show the dependency of the per-unit-length inductance and resistance on the frequency. The input to this circuit is a 40-ns pulse with 2-ns rise and fall times. The proposed algorithm was used to find the sensitivity of the output voltage $V_{\rm out}$

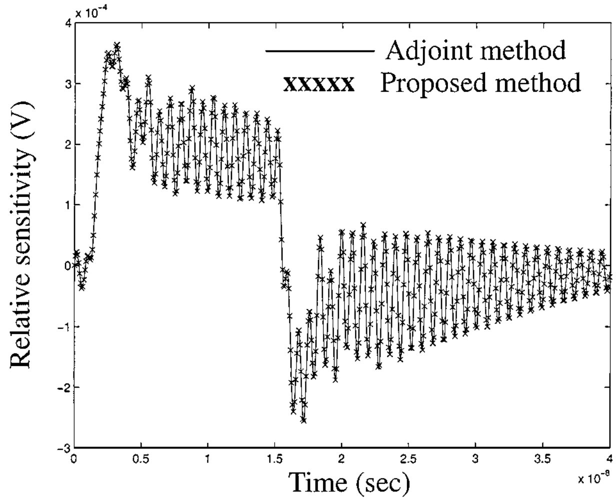

Fig. 8. Relative sensitivity with respect to resistance per unit length.

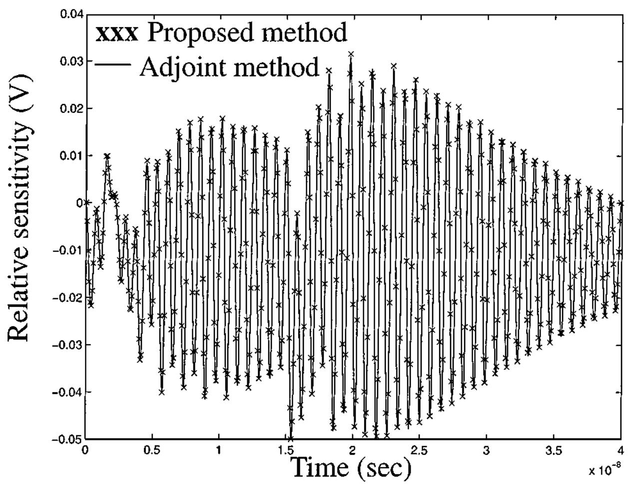

Fig. 9. Relative sensitivity with respect to inductance per unit length.

with respect to the per-unit-length capacitance of transmission line A. The time-domain sensitivity was then found using the fast Fourier transform (FFT). The results obtained using the proposed technique and perturbation method are compared in Fig. 5. No noticeable difference was observed.

Example 2: To demonstrate the efficiency of the method, a relatively large interconnect circuit consisting of 458 resistors, inductors, and capacitors and 12 transmission lines (Fig. 6) is considered. The input pulse and the output response at the node of interest are shown in Fig. 7. The relative sensitivity with respect to the parameters of one of the transmission lines was computed using the proposed algorithm. The time-domain sensitivity was obtained using the FFT and is shown in Figs. 8–11. The response obtained from the proposed reduced-order adjoint technique matched those of the conventional adjoint technique with no noticeable difference. Two expansions were used to match the sensitivity up to 1 GHz. Only two L/U decompositions of the original system and one QR factorization of the subspace were required to obtain the response at 100 frequency points. Note that computation cost of the QR factorization needed to find the congruent transformation is comparable to that of one L/U decomposition. The adjoint technique

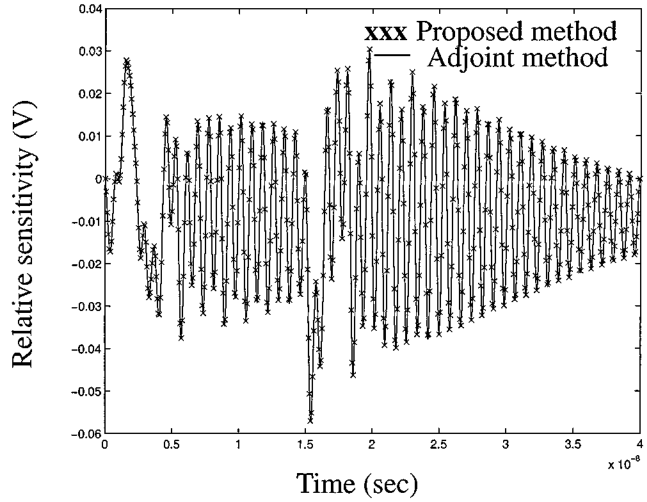

Fig. 10. Relative sensitivity with respect to capacitance per unit length.

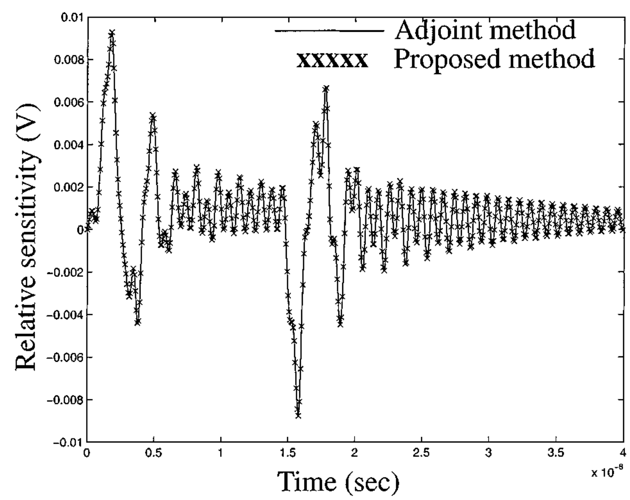

Fig. 11. Relative sensitivity with respect to transmission-line width.

TABLE I
OMPARISON OF THE TOTAL EXPENSE FOR DIFFERENT METHODS

	# of equivalent L/U decompositions	Speed-up ratio
Perturbation	1001	1
Conventional adjoint technique	100	10
Reduced-order adjoint technique	3	334

required 100 L/U decompositions. Table I compares the number of L/U decompositions required for sensitivity analysis with respect to ten different parameters at 100 frequency points.

VI. CONCLUSIONS

An efficient algorithm, based on congruent transformation, for evaluating the frequency- and time-domain sensitivity of large lossy coupled transmission-line networks has been presented in this paper. The algorithm is based on projecting the adjoint network equations on a reduced-order subspace that preserves the circuit moments. Using this algorithm, output sensitivities of large linear networks can be calculated accurately with respect to lumped components and parameters of distributed elements. The proposed algorithm provides a significant decrease in the computational expense for sensitivity analysis.

REFERENCES

[1] M. Nakhla and R. Achar, “Modeling and simulation of high-speed interconnects,” in *Handbook on VLSI*. Boca Raton, FL: CRC Press, 2000, ch. IV.

[2] A. Deustsch, “Electrical characteristics of interconnections for high-performance systems,” *Proc. IEEE*, vol. 86, pp. 315–355, Feb. 1998.

[3] E. Chiprout and M. Nakhla, *Asymptotic Waveform Evaluation and Moment Matching for Interconnect Analysis*. Norwell, MA: Kluwer, 1993.

[4] A. Odabasioglu, M. Celik, and L. T. Pileggi, “PRIMA: Passive reduced-order interconnect macromodeling algorithm,” *IEEE Trans. Computer-Aided Design*, vol. Aug., pp. 645–654, Aug. 1998.

[5] Q. Yu, J. M. L. Wang, and E. S. Kuh, “Passive multipoint moment matching model order reduction algorithm on multiport distributed interconnect networks,” *IEEE Trans. Circuits Syst. I*, vol. 46, pp. 140–160, Jan. 1999.

[6] P. K. Gunupudi, M. S. Nakhla, and R. Achar, “Multi-point multi-port reduction of high-speed distributed interconnects using Krylov-space techniques,” in *Proc. IEEE Int. Circuits Syst. Symp. Dig.*, vol. 6, June 1999, pp. 242–245.

[7] R. W. Freund and P. Feldmann, “Small-signal circuit analysis and sensitivity computations with the PVL algorithm,” *IEEE Trans. Circuits Syst. I*, vol. 43, pp. 577–585, Aug. 1996.

[8] J. Y. Lee, X. Huang, and R. A. Roher, “Pole and zero sensitivity calculation in asymptotic waveform evaluation,” *IEEE Trans. Computer-Aided Design*, vol. 11, pp. 586–597, May 1992.

[9] S. Lum, M. S. Nakhla, and Q. J. Zhang, “Sensitivity analysis of lossy coupled transmission lines with nonlinear terminations,” *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2089–2099, Dec. 1991.

[10] E. Chiprout and M. S. Nakhla, “Analysis of interconnect networks using complex frequency hopping (CFH),” *IEEE Trans. Computer-Aided Design*, vol. 14, pp. 186–200, Feb. 1995.

[11] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*. New York: VNR, 1994.

[12] Y. Saad, *Iterative Methods for Sparse Linear Systems*. Boston, MA: ITP, 1996.

[13] A. E. Ruehli, “Inductance calculations in a complex integrated circuit environment,” *IBM J. Res. Develop.*, vol. 16, pp. 470–481, Sept. 1972.

[14] C. Wei, R. F. Harrington, J. R. Mautz, and T. K. Sarkar, “Multi-conductor transmission lines in multilayered dielectric media,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 439–450, Dec. 1984.

[15] J. Venkatraman, S. M. Rao, A. R. Djordjevic, T. K. Sarkar, and Y. Naiheng, “Analysis of arbitrarily oriented microstrip transmission lines in arbitrarily shaped dielectric media,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 952–959, Oct. 1985.

[16] C. S. Walker, *Capacitance, Inductance and Crosstalk Analysis*. Norwood, MA: Artech House, 1990.

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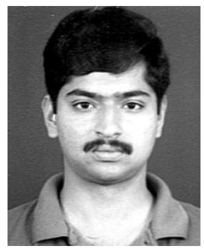

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