Direct Synthesis of Microwave Filters Using Inverse Scattering Transmission-Line Matrix Method

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***Abstract*—This paper proposes a new design procedure for planar microwave filters based on the inversion of the one-dimensional transmission-line matrix (TLM) method. The essence of the technique is the solution of the inverse scattering problem using a TLM-based algorithm instead of using equivalent circuits. The procedure consists of determining the geometry of the obstacle that generates the desired scattered field. In the case of filters, this field is the time-domain input reflection coefficient, and the geometry is the impedance profile of the filter. The procedure was validated with the design of low-pass and bandstop filters. It can be used to create filters with arbitrary characteristics.**

***Index Terms*—Distributed parameter circuits, electromagnetic scattering inverse problems, electromagnetic transient scattering, transmission-line matrix (TLM) methods.**

I. Introduction

The inverse scattering problem deals with the reconstruction of the geometry of an object from its scattered field [1]–[5]. In the case of microwave passive linear devices, this field is the total transmission or reflection coefficient in time or frequency domains. Therefore, this is a synthesis problem where the solution is the reconstruction of the impedance profile of a nonuniform transmission line. Consequently, the desired reflection (or transmission) coefficients of the device determine the profile. In [1], Roberts and Town used an integral equation procedure to solve the inverse scattering problem in the calculation of the impedance profile. The input parameter was the reflection coefficient response of a desired filter. Recently, Le Roy *et al.* developed the continuously varying transmission-line technique [2] to design filters by impedance profile optimization directly from the input reflection coefficient. Therefore, it is possible to use different algorithms to determine the geometry that provides the desired scattered field. This paper uses a modified procedure based on the inversion of the transmission-line matrix (TLM) method. In this case, the solution is exact within the sampling accuracy of the time-domain input reflection coefficient. This approach enhances the applicability of TLM. The proposed method is a synthesis tool for linear passive microwave devices.

II. Theory

The formation process of the input reflection coefficient is the main concept in inverse TLM method. All the discontinuities along the transmission line contribute to the composition of the input coefficient. The problem consists of a wave incident at the input point of a nonuniform transmission line of impedance profile $Z(x)$ (Fig. 1).

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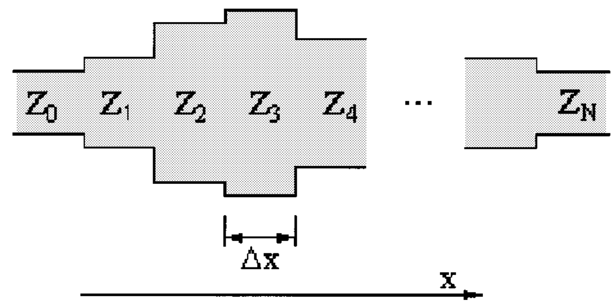

Fig. 1. Discretized nonuniform transmission line.

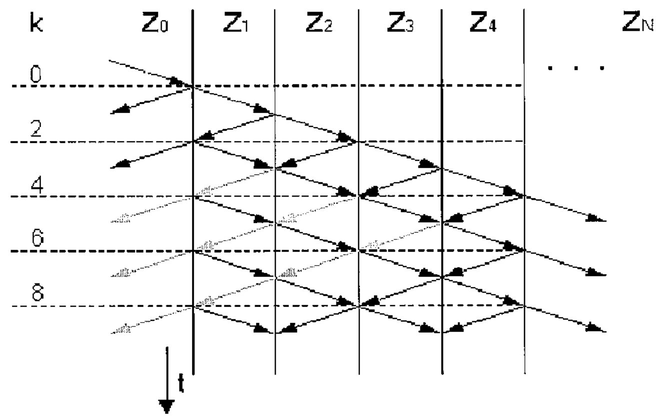

Fig. 2. Space–time diagram of incident and reflected pulses.

In this case, a good approximation of the continuously varying impedance of a nonuniform transmission line is a composition of infinitesimal sections of transmission line with length Δx and impedance $Z(x)$. In the discrete case, the wave propagation process is a result of the infinite sum of the transmitted and reflected waves from each line interface, as shown in the space-time diagram of Fig. 2.

Consequently, in a transmission line of length L with N layers, the time-domain response to an input impulse $\delta(t)$ is the input reflection coefficient

$$\Gamma_i(t) = \sum_{k=0}^{\infty} a_k \delta(t - 2k\Delta t) \tag{1}$$

where a_k is a combination of the reflection coefficient in section $n = (k+2)/2$ of the impedance profile and the transmitted and reflected coefficients from the previous sections. Analyzing the space–time diagram of Fig. 2 and the profile in Fig. 1, it is

TABLE I
VALUE OF a_k GIVEN BY EACH IMPEDANCE LAYER

k	a_k
0	Γ_{01}
2	$T_{01}\Gamma_{12}T_{10}$
4	$T_{01}\Gamma_{12}\Gamma_{10}\Gamma_{12}T_{10} + T_{01}T_{12}\Gamma_{23}T_{21}T_{01}$

possible to obtain a_k as a function of internal transmission and reflection coefficients shown in Table I, where Γ_{kj} and Γ_{kj} represent the reflection and the transmission coefficients between layers k and j.

The mathematical expression of a_k starts to become cumbersome after $k = 4$, as seen in Table I. However, if the time-domain input reflection coefficient is available, one can calculate the reflection coefficient of each interface by knowing the mathematical expression of a_k. The value of a_k yields the reflection coefficient of the interface and the impedance of the particular layer

$$Z_n = Z_{n-1}\frac{1+\Gamma_m}{1-\Gamma_m} \tag{2}$$

where Γ_m is equivalent to the reflection coefficient between layers n and $n-1$ ($\Gamma_{n\ n-1}$).

The problem is that a closed expression of a_k is not simple. It involves not only the impedance layer of interest but also a nonlinear combination of the characteristics of all previous layers. We propose a solution to this problem based in the inversion of the one-dimensional (1-D) TLM method. In the inverse procedure, the TLM method can accurately represent the reflection and transmission of each impedance layer of the structure.

One possible application of the one-dimensional TLM method [6], [7] is the solution the wave equation on transmission lines in one dimension. It discretizes the device in N sections. Each one has a length Δx (given by $\Delta t/v$—where Δt is the timestep and v is the speed of light) and impedance Z_n. The calculation is performed in the time domain, and at each timestep the reflected waves are calculated using

$$\begin{aligned} V_t &= \frac{2}{1+Y_n}(Y_{nk}V_n^i + {}_kV_n^i) \\ {}_kV_n^r &= V_t - {}_kV_n^i \\ {}_kV_n^r &= V_t - {}_kV_n^i \end{aligned} \tag{3}$$

where Y_n is the ratio between admittances of adjacent sections, k is the timestep of the calculated section, and n is the position of the calculated section. The transmitted waves are calculated using

$$\begin{aligned} {}_{k+1}V_n^i &= {}_kV_{n-1}^r \\ {}_{k+1}V_{n+1}^i &= {}_kV_n^r. \end{aligned} \tag{4}$$

This version of TLM is slightly different when compared with traditional implementations of the method. In this case, the theory of method does not rely on the analogy between waves and fields but in the direct application of the various reflections between adjacent sections. Once the simulation is completed, the input reflection coefficient of the structure is readily available. One interesting feature of 1-D TLM is that in some implementations there is no numerical dispersion associated with the numerical algorithm. Since the procedure uses one of such implementations, the calculated response is exact within the sampling limits of the spatial and temporal discretization.

The proposed method uses TLM as a tool to determine the reflection coefficient of each impedance layer of the structure. The inverse TLM algorithm separates all the components of the input reflection coefficient into reflection coefficients of each interface. In order to understand the algorithm, one should consider the reflection from the first three layers of the nonuniform transmission line expressed in Fig. 1. The space–time diagram (Fig. 2) shows the composition of the time-domain reflection coefficient (see also Table I). Consider two different cases in the calculation of the input reflection coefficient from the kth layer ($k = 1, 2$, or 3).

1) In the first case, $Z_k = Z_{k-1}$ and the reflection coefficient Γ_{kk-1} is zero (matched load).
2) In the second case, $Z_k = \infty$ and the reflection coefficient Γ_{kk-1} is one (open circuit load).

If one executes this procedure for each layer using the results from Table I, the input reflection coefficient Γ_i is as follows.

Layer 1) $k = 0$—Interface between Z_0 and Z_1

$$\Gamma_i^{\text{table}} = \Gamma_{01} \tag{3a}$$
$$\Gamma_i^{\text{matched}} = 0 \tag{3b}$$
$$\Gamma_i^{\text{open}} = 1. \tag{3c}$$

Layer 2) $k = 2$—Interface between Z_1 and Z_2

$$\Gamma_i^{\text{table}} = \Gamma_{12}T_{10}T_{01} \tag{4a}$$
$$\Gamma_i^{\text{matched}} = 0 \tag{4b}$$
$$\Gamma_i^{\text{open}} = T_{10}T_{01}. \tag{4c}$$

Layer 3) $k = 4$—Interface between Z_2 and Z_3

$$\Gamma_i^{\text{table}} = \Gamma_{23}T_{10}T_{01}T_{12}T_{21} + \Gamma_{12}T_{10}T_{01} \tag{5a}$$
$$\Gamma_i^{\text{matched}} = \Gamma_{12}T_{10}T_{01} \tag{5b}$$
$$\Gamma_i^{\text{open}} = T_{10}T_{01}T_{12}T_{21}. \tag{5c}$$

From (3a)–(5c), it is possible to reconstruct the general expression for the input reflection coefficient shown in Table I

$$\Gamma_i^{\text{table}}(2k) = \Gamma_{kk-1}\Gamma_i^{\text{open}}(2k) + \Gamma_i^{\text{matched}}(2k). \tag{6}$$

It is simple to perform these calculations for a known structure, but what if the structure is not previously known? In this case, at each even timestep ($2k$), two TLM simulations have to be performed to determine the reflection coefficient. One simulation will be executed assuming the next layer is a matched load. The result will be $\Gamma_i^{\text{matched}}(2k)$. The other simulation will assume that the next layer is an open circuit. The result will be $\Gamma_i^{\text{open}}(2k)$. The combination of (6) with the time-domain reflection coefficient ($\Gamma_i(2k)$) yields the reflection coefficient of layer k, Γ_{kk-1}. Using this coefficient in (2) results in the impedance of the layer. The procedure is repeated recursively. As an example, consider the case of a three-layer unknown structure, such as the one shown in Table I.

1) At $k = 0$, the impedance profile has no elements and the first layer is obtained using (2). There is no need to perform a TLM simulation. The first element of the profile is calculated from the results shown in (3).
2) At $k = 2$, the impedance profile has one element, and two TLM simulations of a single-layer structure (obtained from the result in $k = 0$) are performed. The simulation results shown in (4) are used to calculate the impedance of the second layer.
3) At $k = 4$, the impedance profile has two elements, and two TLM simulations of a double-layer structure (obtained from the results in $k = 0$ and $k = 2$) are performed. The simulation results shown in (5) are used to calculate the impedance of the second layer.

At each timestep $2k$, the impedance profile up to the kth layer is determined using two simulations of a TLM structure with $k-1$ elements. The procedure is repeated until the end of the time-domain input reflection coefficient response. If the response has $2N$ timesteps, then the structure will have N elements.

III. Implementation Issues

The direct synthesis procedure is different from the design based on equivalent circuit approximations. Since it includes a numerically intensive computation, issues of processing time and numerical accuracy are important. It is also necessary to discuss the effects of two-dimensional wave propagation, sampling of the time-domain reflection coefficient, feasible impedance values, and truncation of the filter in the final design.

A. Processing Time

The first point to study in this procedure is the numerical processing time. In the case of a structure with N elements, $2N$ TLM simulations are necessary. The processing time of the algorithm is on the order of $o(N\lfloor(N+1)^2 - N - 1\rfloor)$, as shown in Fig. 3.

If one doubles the number of elements, the computation time will be roughly eight times longer. An optimization of the algorithm will reduce the processing time to $o(N^2)$. This is possible since several calculations are redundant. In the calculation process of the Nth layer of a nonuniform transmission line, some modifications are possible. The use of TLM results from the calculation of the $N-1$ layer avoids extra calculations.

B. Numerical Accuracy

The second point to study is the numerical accuracy. The technique is an inverse scattering procedure. Therefore, once it is applied to the known response of a nonuniform line, it returns the impedance profile of the line. We used random nonuniform lines (the impedance of the line varies randomly with its length) to verify the numerical accuracy of the procedure. The results show that the error of the procedure is very small (10^{-8}%), as shown in Fig. 4, but it grows as the number of sections increases. The truncation and roundoff are the main reasons.

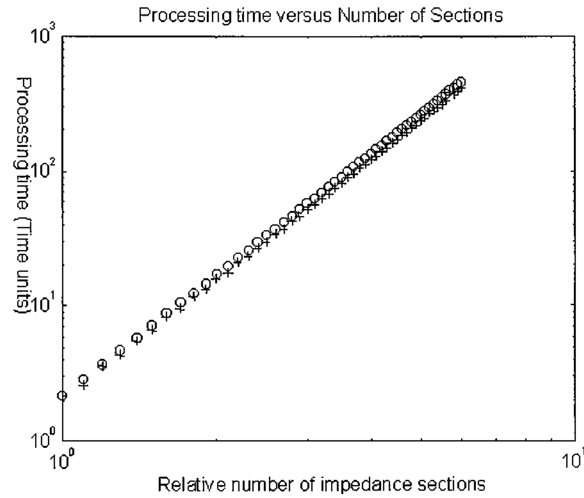

Fig. 3. Computational time and number of sections, calculated (○), and experimental (+).

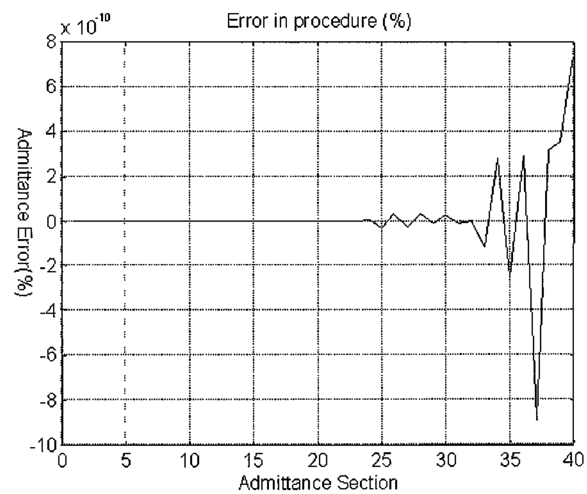

Fig. 4. Percentage error in procedure.

Another factor that influences error is the length and decreasing rate of the time-domain response. The premature truncation of time-domain response can increase the error to higher levels. This shows that inverse scattering TLM is very accurate, although it can be quite time consuming for very large structures.

The signal-to-noise ratio (necessary of accurate impedance profile calculation) has effects in the profile error. This is an important parameter to test if the algorithm can be used to calculated impedance profiles from real data. In real measurements, noise is always present even in reduced levels. In this case, the signal-to-noise ratio (S/N) determines the accuracy of the reconstructed profile. After adding white Gaussian noise to several known time-domain responses, we observed that the S/N needs to be higher than 60 dB (for a 5% error in the profile) for rapidly varying impedance profiles. However, if impedance profile varies slowly (it is composed mainly by a few sections of different impedance), the S/N can be as high as 15 dB for a 5% error. This suggests the use of the algorithm for the calculation of the impedance profile using real data in the slowly varying profile case.

C. Effects of Two-Dimensional Wave Propagation

The inverse scattering algorithm presented in this paper has good performance for one-dimensional wave propagation cases. However, in some cases, it is necessary to study the two-dimensional case. It is possible to adapt the procedure for two-dimensional TLM and use it in different structures. This modification involves some changes in the calculation of the reflection coefficient. The inverse 1-D TLM procedure works well only for TEM waves. Nonetheless, it is possible to modify 1-D TLM for special two-dimensional cases.

If the wave is TEM and is incident at the interface with a certain angle, then it is necessary to modify the algorithm to reflect impedance changes. In this case, the calculated impedance is a function of the incidence angle. The transmitted wave will modify the angle of propagation as it passes through the interface. Therefore, it will be necessary to correct both impedance and angle as the wave propagates. If the propagation is TEM or quasi-TEM, the procedure does not demand significant changes. The input reflection coefficient is extracted from the S-parameters of the structure.

D. Effects of Sampling, Impedance Values, and Truncation

As shown in (3)–(5), the time-domain response of the input reflection coefficient has nonzero elements only at even timesteps. Consequently, in the synthesis procedure, it is necessary to sample the time-domain response of the desired filter characteristic. Since the sampling of the response affects the gain of the filter, it is necessary to normalize the filter response before the method is used. Another effect of this sampling is the duplication of the filter response in the frequency domain. This duplication may cause problems (aliasing) in the design of certain filters. Another implementation issue is that the filters created in this procedure usually have low-pass characteristics. It is mathematically possible to create bandpass or high-pass filters with this method. However, these filters may have unrealizable impedance values. A possible solution is the use of coupled transmission lines in the design suitable bandpass and high-pass filters. This approach is still under study. The last implementation issue regards the length of the structures. In a general case, the time-domain response of the reflection coefficient is infinite. Since the method uses this response to calculate the filter, the resulting structure is a truncated version of the ideal filter. This is quite common if Chebyshev and Butterworth functions are used. If the frequency-domain response is periodic, there is a reduction in the effect of premature filter truncation. Although this may result in spurious passbands, the final length of the filter can be quite compact. Therefore, there is a compromise between accurate frequency response and the dimensions of the structure.

IV. EXPERIMENTAL RESULTS

The procedure was validated with three filter designs. In all cases, a MATLAB code was used to perform the synthesis procedure and generate an impedance profile. The result is an impedance profile in a staircase configuration. If the spatial discretization is small (compared to the wavelength of interest), the interpolation result will not cause problems. However, if this is not true, the interpolation procedure may result in distortion of the desired response. The design procedure is fully automatic for several kinds of filters. The assembled filters were three low-pass filters (two tenth-order Chebyshev and a fifteenth-order Butterworth). The low-pass filters shown in this paper had cutoff frequencies of 5 GHz. In view of the choice of manufacturing technology, one Chebyshev filter needed truncation and interpolation. Since the response of the interpolated filter may not be the same as the staircase one, an analysis procedure is necessary. A TLM algorithm performed the simulation of the interpolated filters. Fig. 5 shows the layout of the interpolated filter. The other filters did not need interpolation (Figs. 6 and 7).

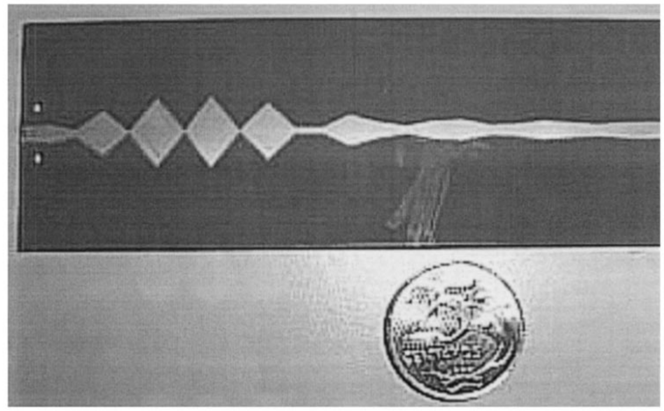

Fig. 5. Layout of the assembled interpolated low-pass filter.

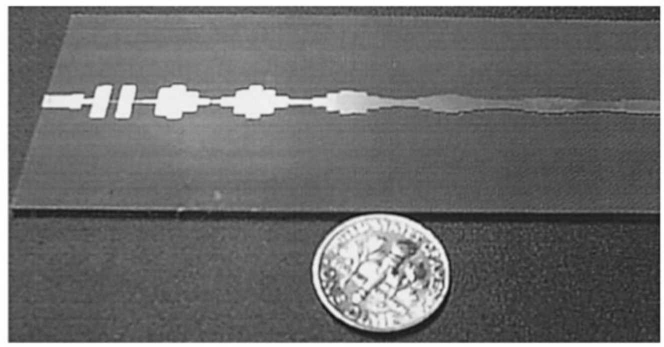

Fig. 6. Layout of the assembled Chebyshev low-pass filter.

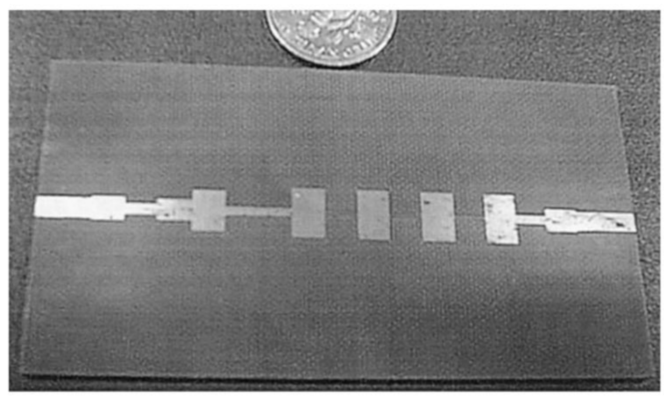

Fig. 7. Layout of the assembled Butterworth low-pass filter.

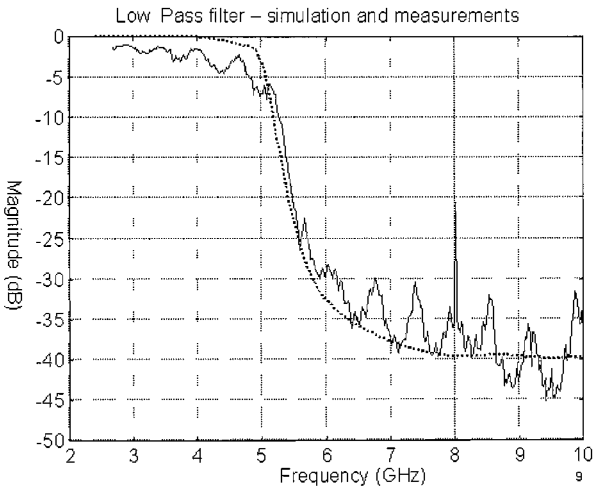

Fig. 8. Measured and simulated results for the interpolated low-pass filter. Measurements—solid line, simulation—dashed line.

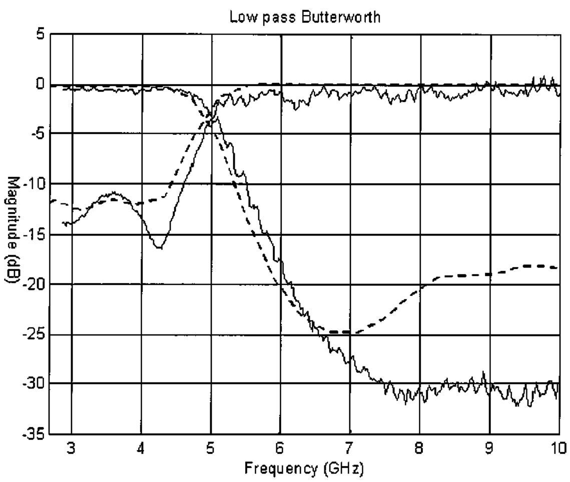

Fig. 9. Measured and simulated results for the Butterworth low-pass filter. Measurements—solid line, simulation—dashed line.

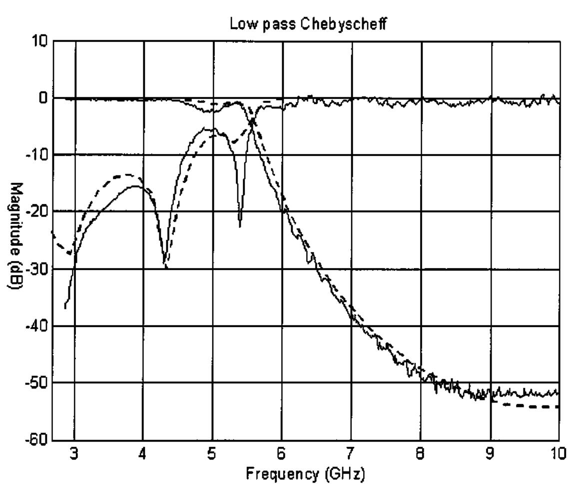

Fig. 10. Measured and simulated results for the Chebyshev low-pass filter. Measurements—solid line, simulation—dashed line.

The filters were built in stripline technology. The substrate had a dielectric constant $\varepsilon_r = 2.17$ and thickness of 1.524 mm. The measurements were executed in the HP 8539 spectrum analyzer. The results are shown in Fig. 8 (interpolated Chebyshev), Fig. 9 (staircase Chebyshev), and Fig. 10 (Butterworth).

The discrepancies in the results of the interpolated band reject filter are a result of the assembly technology. One interesting problem is that the result for the noninterpolated Chebyshev filter has better out-of-band behavior than predicted. The cause is probably the high impedance line obtained in the assembled filter. The error between measured and simulated results is smaller for noninterpolated filters. For the Chebyshev filter, the error was smaller than 1%. The truncation of the filter has some effect on its performance, especially in out-of-band characteristics. However, this is necessary because of the final lengths involved in some design specifications.

V. Conclusions

This paper presented a new synthesis technique for microwave filters using inverse scattering TLM. The procedure consists of using the inverse TLM method as a tool for determining the impedance profile of an unknown microwave structure from its time-domain input reflection coefficient. Filters designed with this technique do not have to use equivalent lumped circuit analysis procedures to achieve the desired response. TLM simulations and experimental measurements validated the procedure. The results show good agreement (within 1% in the Chebyshev low-pass case). Preliminary research activities indicate that it is possible to develop a two-dimensional version of the procedure.

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