Nonlinear Statistical Modeling and Yield Estimation Technique for Use in Monte Carlo Simulations

Jan F. Swidzinski and Kai Chang, *Fellow, IEEE*

***Abstract*—A novel nonlinear statistical modeling technique for microwave devices and a new approach to yield estimation for microwave integrated circuits are presented. The statistical modeling methodology is based on a combination of applied multivariate methods with heuristic techniques. These include principal component analysis and factor analysis in conjunction with maximally flat quadratic interpolation and group method of data handling. The proposed modeling approach, when applied to the database of extracted equivalent circuit parameters (ECPs) for a pseudomorphic high electron mobility transistor device, has proven that it can generate simulated ECPs, S-parameters, that are statistically indistinguishable from a measured ones. A new yield estimation technique based on a Latin hypercube sampling (LHS) is also demonstrated. The LHS-based simulation is utilized as an alternative to primitive Monte Carlo (PMC) simulation in yield analysis. An equally confident yield estimate based on the LHS method requires only one-fourth of those simulations needed when the PMC technique is used.**

***Index Terms*—Algorithms, design automation, microwave field-effect transistors (FETs), modeling, Monte Carlo methods, random number generation, statistical databases, yield estimation.**

I. Introduction

STATISTICAL analysis and design of monolithic microwave integrated circuits (MMICs) requires accurate statistical models of the variation in the active device's performance. Probably the most commonly used approach today for statistical device modeling is based on the dc and S-parameter measurements for a sample of finished devices [1], [2]. Each set of measured dc and S-parameter data, corresponding to one field-effect transistor (FET) device, is converted to the corresponding parameters of the equivalent circuit through a reliable parameter extraction procedure. The statistical properties of the equivalent circuit parameters (ECPs) are then examined, and the estimates of the means (μ), the standard deviations (σ), and the correlation coefficients (ρ) are calculated. Finally, a statistical model based on some multivariate or heuristic techniques capable of recreating those means, standard deviations, and correlations is developed. Our work will follow this approach.

To date three distinctive statistical modeling techniques applied to the FET equivalent circuit parameters have been reported [1]–[6]. The first approach is commonly referred to as a plus-minus sigma $(\pm\sigma)$ model [1] and has been widely accepted in the microwave industry. The plus-minus sigma $(\pm\sigma)$ models, however, are known to be unnecessarily conservative and usually represent physically impossible devices due to inability to account for the correlations that exist between device parameter variations. The second approach is based on principal component analysis (PCA) [2]–[4]. This method orthogonalizes the extracted FET equivalent circuit parameters into a new set of hypothetical variables called principal factors. When using this method, equations that can recreate the original ECPs' correlation structure can then be written in terms of a linear combination of the orthogonal principal factors. Statistical simulations [2] have shown that this approach can accurately predict ECPs' means, standard deviations, and linear correlations. The work of Meehan and Anholt [7], [8], however, provided evidence that linear models fail to recreate the S-parameters from which the ECPs' model was originally extracted. Improvements to statistical device modeling via heuristic methods have been proposed by Bandler [5], [6]. However, the nonlinearities of ECPs as well as their ability to reproduce original S-parameters have also not been addressed adequately. To the best of our knowledge, no nonlinear statistical characterization methodology for microwave devices has been reported.

In this paper, a novel nonlinear statistical modeling technique for equivalent circuit parameters is developed. This technique combines multivariate methods such as PCA [9] and factor analysis (FA) [9] with heuristic algorithms such as maximally flat quadratic interpolation (MFQI) [10] and group method of data handling (GMDH) [11]–[14]. The result of this approach is a very sophisticated statistical model capable of preserving ECPs' (S-parameters) means, standard deviations, correlations, and nonlinear relationships with high accuracy.

The primary goal of statistical modeling is to provide accurate models for yield estimation. Yield is commonly approximated by the primitive Monte Carlo (PMC) method [15]. The advantage of the PMC analysis is that the method is completely general, with no assumptions regarding circuit complexity or the complexity of the input parameter statistics. The accuracy of the PMC yield estimate, however, for a given confidence is a function of the number of trials used to form that estimate. The variance of the estimate varies as the inverse of the square root of the number of trials. Thus, to half the variance in the estimate, a quadrupling of the number of trials is necessary. To date several variance reduction techniques have been investigated, namely, importance sampling [16], stratified sampling [16], and control variates [16]. Hocevar *et al.* [16] have shown that the importance sampling is generally not very effective for variance reduction in a Monte Carlo yield estimation, and the efficiency of the stratified sampling is not significant with respect to the complexity of

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The authors are with Texas A&M University, College Station, TX 77843 USA.

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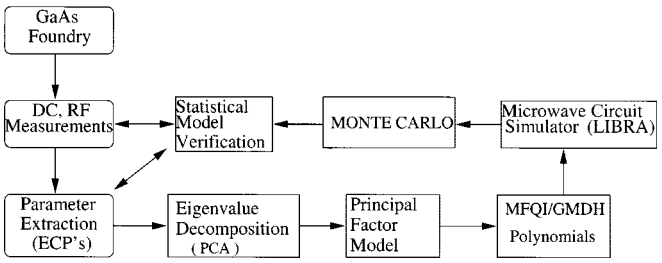

Fig. 1. Statistical modeling methodology.

the implementation. The generality and usefulness of the control variate (shadow model technique) in comparison with PMC method has been confirmed by Hocevar *et al.* [16]. However, this method requires *a priori* information about the circuit behavior, and it only marginally increases the efficiency and accuracy of the yield analysis.

The second contribution of this article is the analysis and novel application of the Latin hypercube sampling (LHS) as an efficient variance reduction method for yield estimation. This mathematical technique was originally pioneered by McKay [17] for one-dimensional space and was restricted to monotonic functions only. The technique was generalized to n-dimensions by Keramat and Kielbasa [18] and applicable to any function. Theoretical results presented in [18] have shown that LHS is equivalent to PMC in its generality, yet the variability of statistical estimators based on the same sample size is significantly reduced.

II. Statistical Modeling Fundamentals

The modeling steps of the proposed statistical methodology are depicted in Fig. 1. In Stage I, statistical properties of ECPs are derived from dc and S-parameter measurements through a reliable extraction procedure. The means, standard deviations, and correlation coefficients are calculated. In Stage II, eigenvalue decomposition (i.e., principal component analysis) is used on the ECPs' correlation matrix as an exploratory tool to determine the minimum number of independent dimensions needed to account for most of the variance in the extracted set of variables (ECPs). Eigenvalues are obtained through diagonalization of the correlation matrix $\sum$. It is known that $\sum$ can be represented as

$$\sum = T\Lambda T^t \tag{1}$$

where

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix} \tag{2}$$

and $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of $\sum$ obtained by solving the following equation:

$$\left(\sum - \lambda I\right) = 0 \tag{3}$$

and T is an orthonormal matrix containing eigenvectors (column vectors) T_i corresponding to λ_i solutions of the equation

$$\left(\sum - \lambda_i I\right) T_i = 0 \tag{4}$$

where I is an identity matrix. In Stage III, a factor model based on FA of the following form is created:

$$X_i = \mu_i + \sigma_i(L_{ij}F_j + \epsilon_i) \tag{5}$$

where

- μ_i and σ_i mean and standard deviation of the original variable, respectively;
- L_{ij} loading coefficient of the ith variable on the jth factor;
- F_j jth common factor with mean zero and unit variance $N(0,1)$.

The number of common factors F_j in the model is equal to the number of selected eigenvalues in Stage II. Random error ϵ_i accounts for the residual variance of the ith variable. In most practical cases, the ϵ_i term is small and can be neglected. Such an initial factor model is commonly not easily interpretable, and it is the usual practice to mathematically rotate the L_{ij} loading coefficients until a simpler model structure is achieved. From a mathematical viewpoint, it is immaterial whether the L_{ij} or the rotated L^*_{ij} matrix is used in the model equation [(5)]. The rationale of the rotation procedure is very much akin to sharpening the focus of a microscope in order to see detail more clearly. Ideally, we would like to see a pattern of loadings (L_{ij}) such that each original variable loads highly on a single common factor (F_j) and has small to moderate loadings on the remaining factors. It is not always possible to get this simple structure, however; for most types of data, rotations improve model structure/interpretation significantly [9].

When a factor model with a simple structure is obtained, a more physical model can be constructed. In Stage IV of our methodology, some of the original variables (those with the highest loading coefficients) are selected as the regressors (independent predictors) and the rest as regressands (dependent parameters). There are several reasons why we need to build such a physical model.

1) The process engineers and microwave circuit designers feel more comfortable with the original variables.
2) The nonlinear relationships between parameters can be accounted for by higher order regressions/polynomials.
3) The regression/polynomial models can be used to speed up the characterization of the process/device by its prediction feature.

In general, linear and/or quadratic regression models of the following form might have been adequate:

$$f(x) = \beta_o + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} x_i x_j + \dots + \epsilon \tag{6}$$

where x's and βs represent independent predictors and regression coefficients, respectively. The regression methods, however, have severe limitations.

1) Linear models are not sufficiently accurate.
2) Quadratic models require a large number of collected data (e.g., if the number of variables $n = 40$, then the number of samples needed to build the model is $m = \binom{n+2}{2} =$

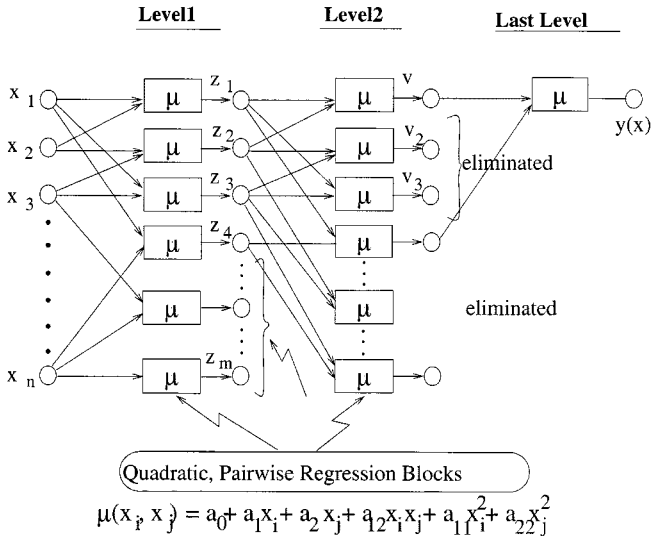

Fig. 2. GMDH modeling strategy.

$(n+1)(n+2)/2 = 861$); additionally, these models are valid only within small exploration region.

3) Higher order regressions often lead to a severely ill-conditioned system of equations.

To alleviate problems of conventional regressions, a combination of heuristic methods MFQI [10] and GMDH [11]–[14] has been utilized in our methodology. The MFQI technique approximates the performance function of interest by interpolating polynomials of the following form:

$$f(x) = a_o + \sum_{i=1}^{n} a_i x_i + \sum_{i=1:j\geq 1}^{n} a_{ij} x_i x_j + \cdots \qquad (7)$$

where x's and a's represent independent predictors and polynomial coefficients, respectively. The MFQI approach is more efficient and more accurate than the traditional quadratic regression [10] methods. Model accuracy, however, just like in regressions, is confined to a small region of parameter space. To overcome that deficiency, the ECPs' data set is divided into k number of overlapping subsets $S_1, \ldots, S_k$ and k MFQI models are created first. These models generate new variables, which are used as inputs to the GMDH algorithm as depicted in Fig. 2.

The GMDH algorithm is a multilayer approximation technique with a neural network-type architecture, proposed in [11]–[13], to model the input and output relationship of a complex system. At each layer, new generations of complex equations are constructed from simple forms. The survival-of-the-fittest principle (appropriate thresholds) determines the equations that are passed on to the next layer and those that are discarded, that is, only the best combination of input parameters is allowed to pass through to the next layer. The model obtained after each layer is progressively more complex than the models excluded during the preceding layers. To avoid an overfit, the GMDH algorithm divides the data sample into a) the training set, which is used for generation of several competing alternative models, and b) the checking set, which is used to check the accuracy of the models generated and for the selection of the best models at each layer. This provides the self-organizing feature of the algorithm, leading to models of optimal complexity. The number of modeling layers is increased until the newer models begin to have poorer powers of predictability than did their predecessors. The net result is a very sophisticated model from a very limited data set. The resulting modeling polynomial has the form

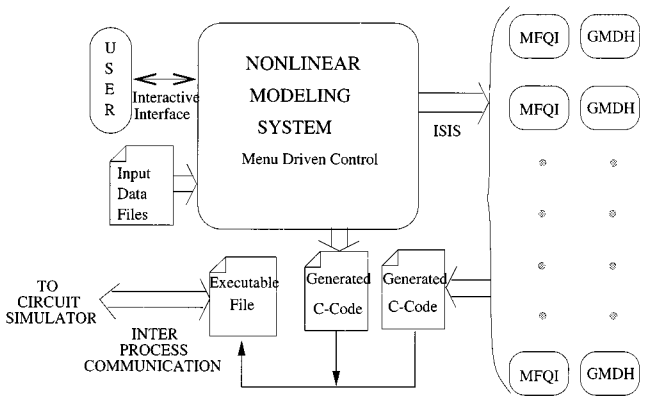

Fig. 3. Nonlinear modeling system—computer implementation.

$$f(x) = a_o + \sum_{i=1}^{n} b_i x_i + \sum_{i=1:j=1}^{n} c_{ij} x_i x_j + \sum_{i=1:j=1:k=1}^{n} d_{ijk} x_i x_j x_k + \cdots \qquad (8)$$

where x's and $a_o, b_i, c_{ij}, d_{ijk}$ are independent predictors and polynomial coefficients, respectively.

The combined structure of the MFQI and GMDH methods allows the MFQI algorithm to create several locally accurate and overlapping interpolating models over the entire parameter space first. Then the GMDH algorithm combines these models in an optimal way, based on approximation strategy—minimizing average error. The resulting final model overcomes the limitations of the traditional regressions and is more accurate than the models created by the two individual algorithms. Computer implementation of the composite modeling (MFQI+GMDH) strategy is depicted in Fig. 3.

The final stage (Stage V) validates the generated model by Monte Carlo simulation and comparison of the simulated and original ECPs or by comparison of the simulated and original S-parameter databases.

III. Statistical Modeling Example

The proposed modeling was applied to the database of extracted equivalent circuit parameters for a pseudomorphic high electron mobility transistor (pHEMT) FET device. The lumped element equivalent circuit for pHEMT is shown in Fig. 4.

Thirteen ECPs for each pHEMT device from 27 wafers and six different lots were extracted and recorded [19]. The original ECPs' means, standard deviations, and correlations are given in Tables I and II, respectively.

Initially, we performed exploratory analyses on the original ECPs' correlation matrix utilizing eigenvalue decomposition in order to determine the minimum number of dimensions needed to account for most of the variance in the original set of variables. Eigenvalue decomposition resulted in the vector of eigenvalues and the vector of cumulative explained variance as presented in Table III.

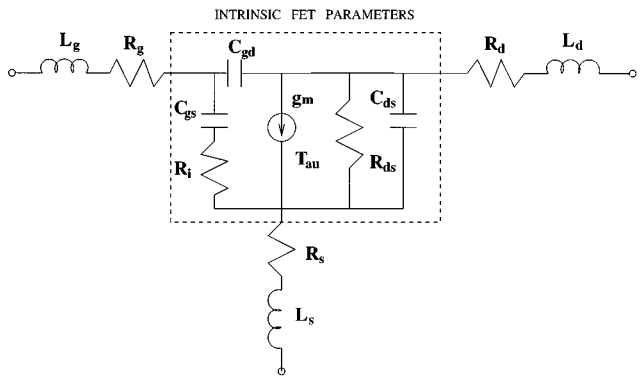

Fig. 4. Equivalent circuit model for pHEMT device.

TABLE I
ORIGINAL ECP MEANS AND STANDARD DEVIATIONS FOR THE pHEMT

Parameter Name	Mean	Standard Deviation
$g_m(S)$	9.493e-2	6.324e-3
$R_{ds}(\Omega)$	373.77	47.39
$C_{gs}(pF)$	0.56	0.0553
$C_{ds}(pF)$	6.556e-2	3.587e-3
$C_{gd}(pF)$	2.243e-2	2.744e-3
$R_i(\Omega)$	3.436	0.451
$T_{au}(pS)$	4.473	0.335
$R_g(\Omega)$	0.575	0.196
$R_d(\Omega)$	0.395	0.176
$R_s(\Omega)$	0.682	0.184
$L_g(nH)$	1.786e-2	7.568e-3
$L_d(nH)$	7.018e-3	2.074e-3
$L_s(nH)$	2.197e-3	1.083e-3

TABLE II
ORIGINAL ECP CORRELATIONS FOR THE pHEMT DEVICE

Var.	Correlations
g_m	1.0
R_{ds}	-0.03 1.0
C_{gs}	0.58 0.66 1.0
C_{ds}	-0.39 -0.37 -0.54 1.0
C_{gd}	0.12 0.93 0.69 -0.36 1.0
R_i	-0.55 0.45 -0.14 -0.06 0.33 1.0
T_{au}	-0.17 0.86 0.60 -0.29 0.77 0.64 1.0
R_g	0.31 -0.08 0.16 -0.22 0.00 -0.76 -0.45 1.0
R_d	-0.21 0.03 -0.28 0.37 0.03 0.18 -0.07 -0.12 1.0
R_s	0.40 0.15 0.51 -0.16 0.06 -0.16 0.16 -0.05 -0.20 1.0
L_g	0.31 -0.63 -0.30 0.01 -0.50 -0.66 -0.75 0.51 -0.04 -0.35 1.0
L_d	0.26 -0.09 0.30 -0.31 -0.18 -0.13 0.10 -0.08 -0.60 0.54 -0.17 1.0
L_s	0.61 -0.04 0.49 -0.38 0.03 -0.66 -0.16 0.46 -0.37 0.29 0.46 0.25 1.0

Note that five eigenvalues are sufficient to explain approximately 90% variance of the original data. This decision is somewhat subjective; as a rule of thumb, avoid eigenvalues that are much smaller than one (Kaiser criterion [9]) or whose individual contributions to the cumulative variance are less than 5%. Based on the above result, a factor model with five common factors utilizing commercial statistical software S-PLUS [20] was developed. To improve interpretibility of the factor model, the "varimax" orthogonal rotation was applied, leading to the pattern of loading coefficients shown in Table IV.

Physically meaningful linear and nonlinear models were built by substituting the hypothetical factors $(F_1, \ldots, F_5)$ with the original variables as independent predictors (those strongly related to F_j). In our case, we selected $(R_{\rm ds}, R_g, g_m, R_d, R_s)$, while the remaining parameters $(C_{\rm gs}, C_{\rm ds}, C_{\rm gd}, R_i, T_{\rm au}, R_g, L_d, L_s)$ are determined in terms of the selected predictors. Based on this selection, linear and

TABLE III
EIGENVALUE DECOMPOSITION OF THE pHEMT ECPS

Parameter	Eigenvalue	Cumulative Explained Variance %
λ_1	4.35	33.47
λ_2	3.79	62.66
λ_3	1.80	76.50
λ_4	1.05	84.58
λ_5	0.65	89.63
λ_6	0.49	93.40
.	.	.
.	.	.
.	.	.
λ_{13}	0.001	100.00

TABLE IV
ROTATED FACTOR MODEL

Var.	F1	F2	F3	F4	F5
g_m	0.026	0.275	0.880	0.119	0.244
R_{ds}	0.983	-0.082	-0.033	-0.02	0.021
C_{gs}	0.716	0.154	0.513	0.231	0.349
C_{ds}	-0.393	-0.059	-0.383	-0.471	0.116
C_{gd}	0.951	-0.014	0.119	-0.096	-0.063
R_i	0.373	-0.84	-0.343	-0.018	-0.208
T_{au}	0.834	-0.428	-0.078	0.138	0.097
R_g	-0.007	0.879	0.143	0.055	-0.142
R_d	-0.01	-0.112	-0.104	-0.852	-0.073
R_s	0.126	-0.012	0.248	0.194	0.798
L_g	-0.586	0.458	0.414	-0.005	-0.472
L_d	-0.093	-0.074	0.088	0.742	0.494
L_s	0.013	0.495	0.590	0.299	0.081

TABLE V
ESTABLISHED ACCURACY: ORIGINAL VERSUS MODEL

Parameter	Orig.(μ)	Est(μ)	Error%	Orig.(σ)	Est(σ)	Error%
$g_m(S)$	9.493e-2	9.50e-2	0.07	6.324e-3	6.505e-3	2.86
$R_{ds}(\Omega)$	373.77	373.95	0.05	47.39	48.54	2.43
$C_{gs}(pF)$	0.56	0.558	0.35	0.0553	0.0517	6.51
$C_{ds}(pF)$	6.556e-2	6.561e-2	0.07	3.587e-3	3.60e-3	0.36
$C_{gd}(pF)$	2.243e-2	2.250e-2	0.31	2.744e-3	2.64e-3	3.79
$R_i(\Omega)$	3.436	3.448	0.35	0.451	0.423	2.8
$T_{au}(pS)$	4.473	4.460	0.29	0.335	0.310	7.46
$R_g(\Omega)$	0.575	0.578	0.52	0.196	0.203	3.57
$R_d(\Omega)$	0.395	0.394	0.25	0.176	0.178	1.14
$R_s(\Omega)$	0.682	0.679	0.44	0.184	0.188	2.17
$L_g(nH)$	1.786e-2	1.751e-2	1.96	7.568e-3	6.885e-3	9.02
$L_d(nH)$	7.018e-3	6.910e-3	1.54	2.074e-3	1.897e-3	8.53
$L_s(nH)$	2.197e-3	2.205	0.36	1.083e-3	1.10e-3	1.57

TABLE VI
MODELED CORRELATIONS FOR THE pHEMT DEVICE

Var.	Modeled Correlations
g_m	1.0
R_{ds}	-0.02 1.0
C_{gs}	0.49 0.64 1.0
C_{ds}	-0.27 -0.28 -0.37 1.0
C_{gd}	0.08 0.88 0.45 -0.24 1.0
R_i	-0.64 0.29 -0.23 -0.15 0.37 1.0
T_{au}	-0.25 0.88 0.47 -0.18 0.77 0.60 1.0
R_g	0.25 -0.04 0.22 -0.30 0.01 -0.70 -0.43 1.0
R_d	-0.17 0.04 -0.31 0.46 0.14 0.22 -0.01 -0.12 1.0
R_s	0.36 0.14 0.63 -0.27 -0.09 -0.19 0.16 -0.04 -0.14 1.0
L_g	0.33 -0.72 -0.42 0.11 -0.61 -0.63 -0.91 0.48 -0.12 -0.24 1.0
L_d	0.32 -0.03 0.55 -0.39 -0.24 -0.18 0.03 -0.20 -0.80 0.64 -0.11 1.0
L_s	0.59 -0.06 0.44 -0.34 -0.08 -0.63 -0.28 0.45 -0.30 0.29 0.43 0.32 1.0

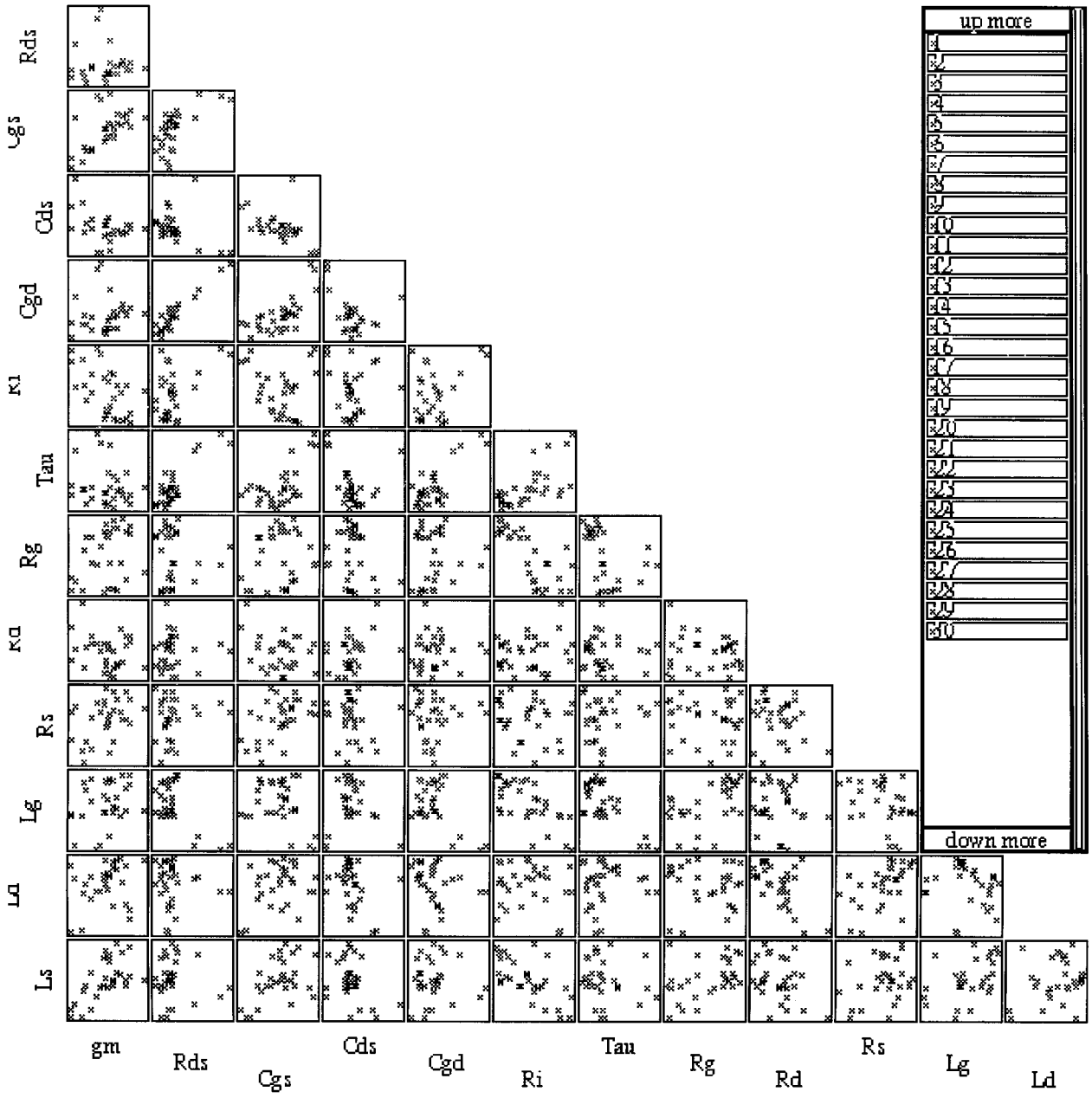

Fig. 5. Scatter plot of original ECPs.

nonlinear physically meaningful polynomial equations, utilizing a combination of the MFQI and GMDH algorithms, were developed. For better efficiency, equations were implemented in C language. Three hundred Monte Carlo simulations on the generated statistical ECP model were performed, which resulted in means, standard deviations, and a correlation matrix as presented in Tables V and VI, respectively.

Maximum errors for the calculated means and standard deviations are 2% and 9%, respectively. The original and modeled linear correlations have also matched very satisfactorily. The most significant achievement of this technique, however, is the accurate reproduction of the nonlinear relationships existing among original ECPs, as depicted in Figs. 5 and 6.

To improve the viability of the proposed methodology, a developed pHEMT statistical device model was implemented in a LIBRA [21] simulator. A pHEMT device was biased with the same settings at which S-parameters were originally extracted. The model was simulated 300 times in the LIBRA simulator. At a randomly chosen frequency (6 GHz in our case), S-parameters were extracted and compared to S-parameters as generated by the original ECPs' database at the same frequency. Linear and nonlinear relationships existing among original S-parameters were reproduced with high accuracy, as depicted in Fig. 7.

Once the reliable statistical equivalent circuit device model has been obtained, the analysis and optimization of any subsystem (power amplifier, mixer, oscillator, etc.) containing the transistor can be made with the aid of common commercial microwave simulators such as LIBRA. The results of such simulation, however, will be questionable for these nonlinear circuits since our statistical models are based on dc and small signal

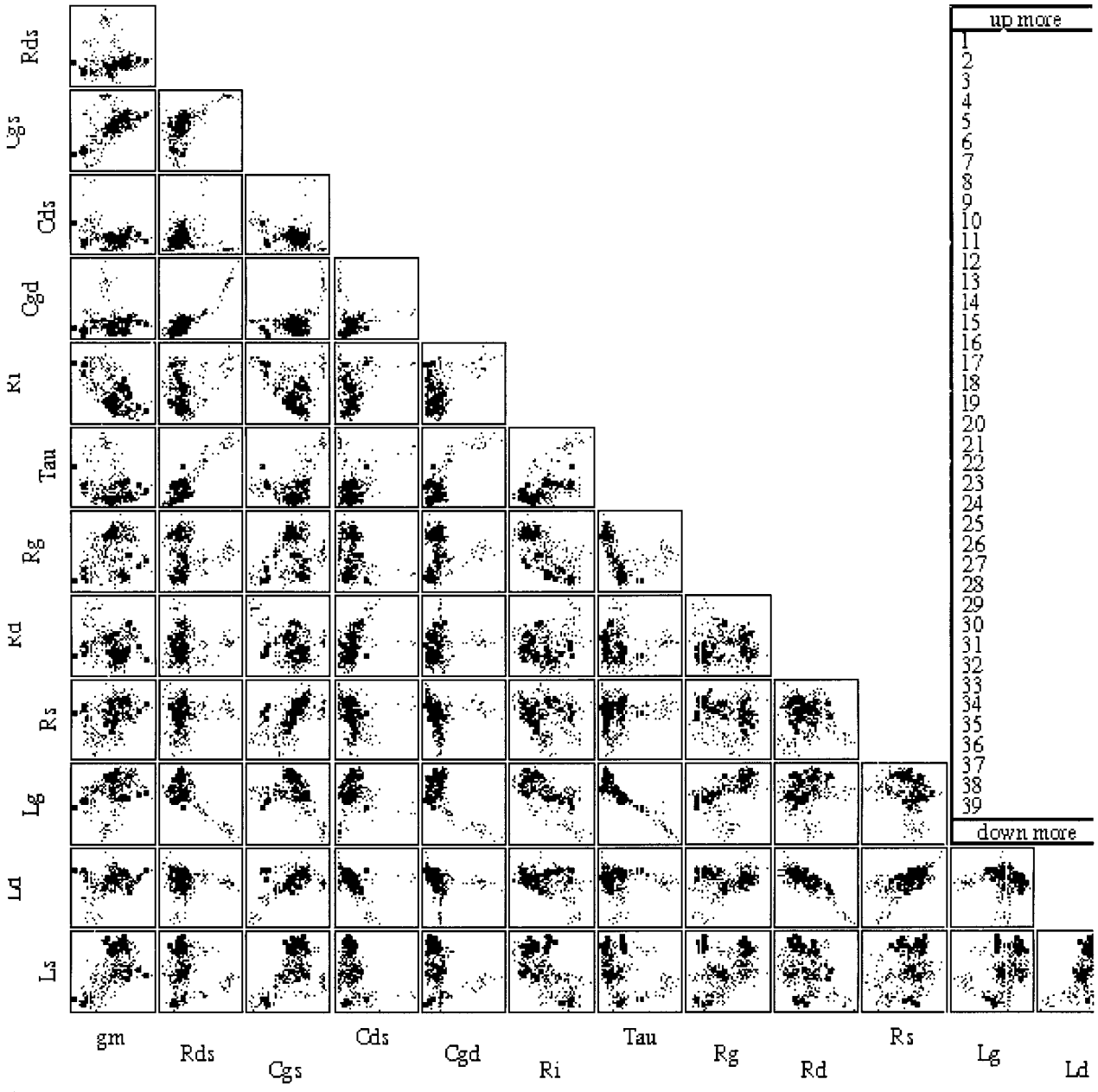

Fig. 6. Scatter plot of modeled ECPs.

measurements under specific bias conditions. Intrinsic parameters of the equivalent circuit model $(g_m, C_{\rm gd}, C_{\rm gs}, C_{\rm ds}, R_{\rm ds})$ are bias (V_g, V_d) dependent. This is difficult and yet continues to be unresolved. The nonlinear statistical device model, which accounts for the voltage dependence of intrinsic parameters, is the subject of further research. We will, however, address one of the avenues we are pursuing. The values of intrinsic elements of Fig. 4 for different gate and drain voltages can be obtained by the S-parameters of the FET in a wide range of bias conditions and extracting a small signal equivalent circuit at each bias point. Intrinsic elements are then expressed as functions of the voltages V_g, V_d. The statistical characterization should be done then in two fundamental stages: characterization 1) due to manufacturing tolerances and 2) due to different biasing conditions. The interpolation method can be utilized, which shifts the statistical characteristics of the former based on specific conditions of the latter (superposition). This approach has the advantage that even with simple interpolation functions, the solution will always be accurate in the bias point. For large amplitude excursions from the bias point, at the extreme voltage values, the accuracy will be poorer, but that should be a secondary effect not affecting the global accuracy of the solution.

IV. Random Numbers—LHS Versus PMC Sampling

Statistical modeling is a required prerequisite but is not sufficient for accurate yield estimation. Yield is commonly approximated by the PMC method. The success of yield estimation based on the Monte Carlo calculation often stands or falls with the quality of the random samples that are used. Random

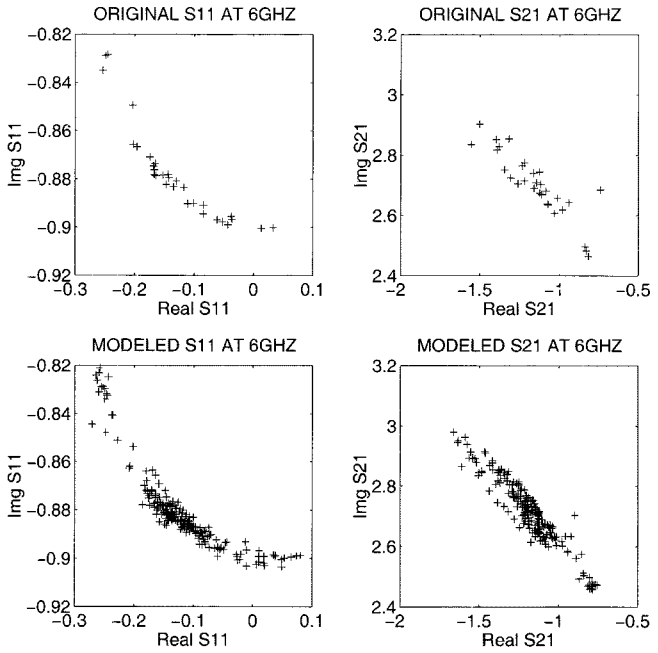

Fig. 7. Original and modeled S_{11} and S_{21} parameters at 6 GHz for pHEMT device.

number generation based on LHS is advocated over traditional industry-standard PMC sampling.

LHS is similar to PMC sampling except in sample generation. LHS operates as follows. Generate a sample size N from the n variables $\Psi = \psi_1, \psi_2, \ldots, \psi_n$ with the joint probability density function (jpdf) of $f_\psi(\Psi)$. The range of each variable is partitioned into N nonoverlapping intervals on the basis of equal probability size $1/N$. One value from each interval is selected at random with respect to the probability density in the interval. The N values thus obtained for Ψ_1 are paired in a random manner with the N values of Ψ_2. These N pairs are combined in a random manner with the N values of Ψ_3 to form N triplets, and so on, until a set of N n-tuples is formed. This set of n-tupels is the Latin hypercube sample. It should be evident from the above description that the LHS generator partitions the disturbance space of interest into N^n cells and samples one point from each cell. This unique sampling scheme is computationally efficient, can cope with many input variables, and ensures that all portions of the range of each input variable is represented.

To quantify the quality of the LHS uniform sample by the scheme described above, a uniform two-dimensional sample of 500 has been generated. The two-dimensional uniform (0,1) plane was divided by m subintervals, such that m by m subregions were created. The error function, which determines the quality of uniform distribution, was defined as in [22]

$$\text{err} = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} \left| \frac{P_{ij} - N\Phi_{ij}}{N\Phi_{ij}} \right| \tag{9}$$

where

N	sample number;
P_{ij}	sample number of two-dimensional random vector in subregion (i,j);
Φ_{ij}	probability of two-dimensional standard uniform distribution in subregion (i,j).

TABLE VII
COMPARISON OF THE DIFFERENT RANDOM NUMBER GENERATORS
(Error ideal $= 0, \max = 1$)

Uniform sample generation	Error
Linear Congruential method[22]	0.571
Square Choosing Middle Method[22]	0.581
Shuffling Method[22]	0.223
Data Coding Method[22]	0.225
LHS - Proposed Method	0.218

The error value was calculated for the sample size $N = 500$ and $m = 10$ (as discussed in [22]) and contrasted with the values reported in [22] for four uniform pseudorandom number generators that are widely used in microwave circuits analysis and optimization (Table VII). LHS sampling displays the smallest error: meaning, it has the best uniform coverage.

Uniform distributions, however, are hardly used in practice, but they serve as a basis for creation of other desirable distributions (Gaussian, for example). Transformation techniques from uniform to Gaussian have been studied in [23]. The most commonly used transformations in today's microwave circuit simulators are:

1) polar form of the Box–Muller method;
2) functional approximation of the inverse transform;
3) lookup table.

These techniques take a uniform sample as input and generate standard normal (Gaussian) variable as output. Any good quality uniform random number generator (i.e., shuffling method, data coding method, etc.) subject to one of the above transformations would give us a Gaussian pdf for a small sample size $N = 50$, as depicted in Fig. 8(a) (shuffling random number generator and polar form of Box–Muller transformation have been utilized in this case). Skewness, poor uniformity, and a clustering of sample points for such small sample sizes are well-known and -understood phenomena [24] in PMC sampling. These detrimental patterns change significantly from one simulation to another when such small samples are used. Consequently, large variability and low confidence in statistical estimators are expected.

The proposed LHS sampling strategy in conjunction with the inverse transform method [18] would give us a pdf for the same sample size $N = 50$ as depicted in Fig. 8(b).

The unique sampling of LHS assures consistent and uniform coverage compared to the sparse or clustered coverage achieved by the PMC technique. The significance of that will be evident in variability reduction and repeatability of yield estimators when a small number of Monte Carlo simulations in yield analyses are used.

V. QUALITY OF YIELD ESTIMATE—LHS VERSUS PMC SAMPLING

The efficiency of LHS over PMC sampling in yield analysis is demonstrated on a 3.8–4.2 GHz single-stage FET amplifier [24]. The circuit schematic is depicted in Fig. 9, and design specifications [24] are as follows: $S_{21} = 15 \pm 1$ dB, $S_{11} \leq -8$ dB, and $S_{22} \leq -8$ dB for frequencies from 3.8 to 4.2 GHz.

The matching circuit elements, namely, $Z_{\text{in}} = 38.15\ \Omega$, $E_{\text{in}} = 85^o$, $L_{\text{in}} = 3.23$ nH, $Z_{\text{out}} = 86\ \Omega$, $E_{\text{out}} = 99.8^o$,

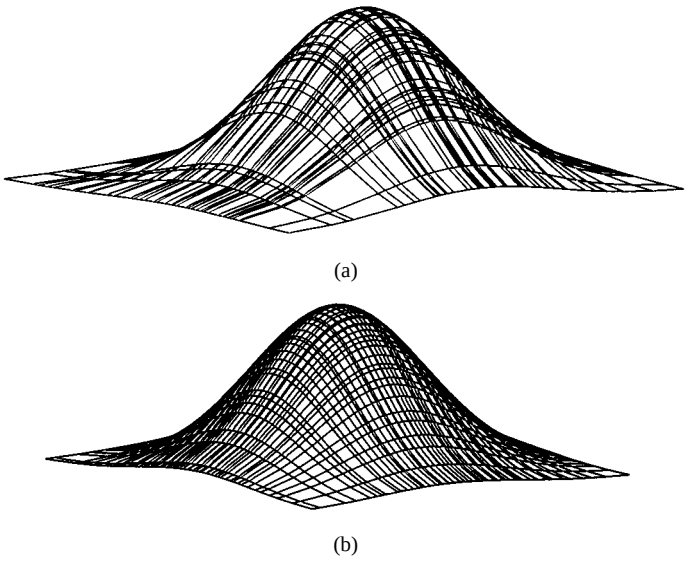

Fig. 8. Two-dimensional Gaussian probability density functions based on (a) PMC and (b) LHS sampling.

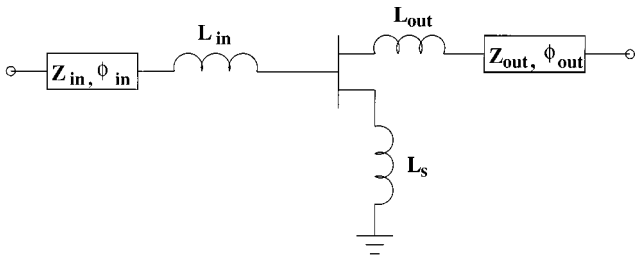

Fig. 9. Schematic of the 3.8–4.2 GHz single stage FET amplifier.

$L_{\text{out}} = 8.48$ nH, and $L_s = 0.57$ nH, were chosen as design variables. Tolerance of 10% was assumed for all components. A statistical device developed in Section III was incorporated into the design so the full characterization of the circuit was completed. A microwave circuit simulator LIBRA [21], in conjunction with the Matlab program, was employed to perform Monte Carlo simulation utilizing both sampling strategies. Sample sizes from ten up to 10 000 based on the LHS and PMC methods were generated. Circuit analyses were performed, and yields for each sample were computed. Fig. 10 summarizes the yield outcomes as a function of sample size for both sampling schemes.

It is evident that the LHS yield estimator displays a stable value for sample sizes of 200 and less, while samples of nearly 1000 were needed for an equally confident yield estimate when the PMC method was used. Our simulation analyses have consistently shown that on average, yield estimate can be obtained with the same confidence by an LHS method, using approximately one-fourth of the simulations usually required by the PMC technique. To improve the viability of this conclusion, ten Monte Carlo iterations, each simulation starting with a different random seed number, were performed for both sampling methods. Equal sample sizes of 200 each time were generated, and yields were plotted as depicted in Fig. 11.

The same experiment was repeated for the PMC method when each sample size was 1000; these results are also plotted in Fig. 11 (as PMC2). Note that approximately 200 LHS versus

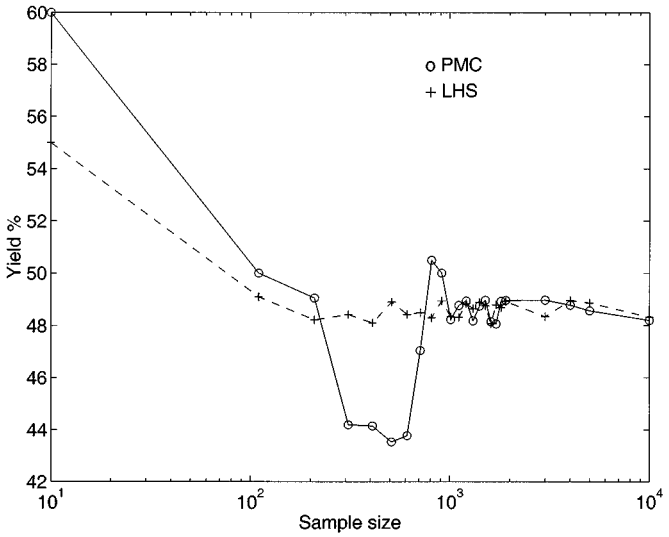

Fig. 10. Variability of yield estimate versus sample size for 3.8–4.2 GHz amplifier based on PMC and LHS method.

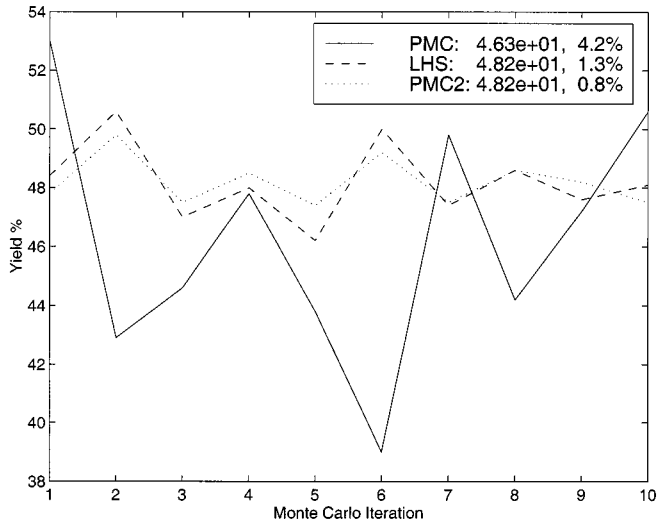

Fig. 11. Estimated yield variability for ten Monte Carlo iterations: PMC and LHS yields are based on samples of 200 and PMC2 yields based on samples of 1000.

1000 PMC simulations were needed to get the same small variability and the same degree of confidence in the yield estimate.

VI. Conclusion

A novel nonlinear statistical characterization methodology for FET equivalent circuit parameters has been presented. The technique is based on a combination of hierarchically structured applied multivariate methods such as principal component analysis and factor analysis with heuristic techniques such as maximally flat quadratic interpolation and group method of data handling. As a result, a very sophisticated model capable of recreating ECPs' means, standard deviations, correlations, and linear and nonlinear relationships is obtained. Model equations, however, are of high complexity and cannot easily be implemented into existing commercial microwave computer-aided design software. An interface between model and microwave circuit simulator is required.

A new approach to yield estimation for microwave integrated circuits based on Latin hypercube sampling also was presented. This sampling method ensures that each of the input variables has all portions of its range represented during simulation irrespective of the sample size. Comparisons of yield estimates based on LHS and PMC sampling have shown that for an equally confident yield estimator, the LHS method on average requires one-fourth the analyses normally used by the PMC technique. The proposed technique is very simple and should be easily integrated into the code of existing microwave circuit simulators.

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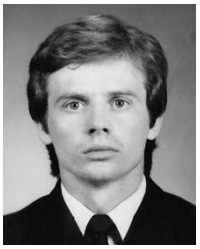

Jan F. Swidzinski received the B.S. degree in applied mathematics from Technical Univeristy Poland in 1988 and the M.S. degree in electrical engineering from Texas A&M University, College Station, in 1997. Currently, he is pursuing the Ph.D. degree at Texas A&M.

His doctoral work is in the area of design, modeling, and characterization of radio-frequency integrated circuits. His particular interests are low-noise amplifiers, mixers, oscillators, transmitters, and receivers.

Kai Chang (S'75–M'76–SM'85–F'91) received the B.S.E.E. degree from National Taiwan University, Taipei, Taiwan, R.O.C., in 1970, the M.S. degree from the State University of New York at Stony Brook in 1972, and the Ph.D. degree from the University of Michigan, Ann Arbor, in 1976.

From 1972 to 1976, he was with the Microwave Solid-State Circuits Group, Cooley Electronics Laboratory, University of Michigan, as a Research Assistant. From 1976 to 1978, he was with Shared Applications, Inc., Ann Arbor, where he worked in computer simulation of microwave circuits and microwave tubes. From 1978 to 1981, he was with the Electron Dynamics Division, Hughes Aircraft Company, Torrance, CA, where he was involved in the research and development of millimeter-wave solid-state devices and circuits, power combiners, oscillators, and transmitters. From 1981 to 1985, he was with TRW Electronics and Defense, Redondo Beach, CA, as a Section Head, developing state-of-the-art millimeter-wave integrated circuits and subsytems including mixers, VCOs, transmitters, amplifiers, modulators, upconverters, switches, multipliers, receivers, and transceivers. He joined the Electrical Engineering Department of Texas A&M University, College Station, in 1985 as an Associate Professor and became a Professor in 1988. In 1990, he became E-Systems Endowed Professor of Electrical Engineering. His current interests are in microwave and millimeter-wave devices and circuits, microwave integrated circuits, integrated antennas, wideband and active antennas, phased arrays, microwave power transmission, and microwave optical interactions. He is an author or coauthor of several books, including *Microwave Solid-State Circuits and Applications* (New York: Wiley, 1994), *Microwave Ring Circuits and Antennas* (New York: Wiley, 1996), *Integrated Active Antennas and Spatial Power Combining* (New York: Wiley, 1996), and *RF and Microwave Wireless Systems* (New York: Wiley, 2000). He was the Editor of the four-volume *Handbook of Microwave and Optical Components* (New York: Wiley, 1989 and 1990). He is the Editor of the *Microwave and Optical technology Letters* and the Wiley book series in "Microwave Optical Engineering." He has published more than 350 technical papers and several book chapters in the areas of microwave and millimeter-wave devices, circuits, and antennas.

Dr. Chang received the Special Achievement Award from TRW in 1984, the Halliburton Professor Award in 1988, the Distinguished Teaching Award in 1989, the Distinguished Research Award in 1992, and the TEES Fellow Award in 1996 from the Texas A&M University.