D-instanton partition functions

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Duality arguments are used to determine *D*-instanton contributions to certain effective interaction terms of type II supergravity theories in various dimensions. This leads to exact expressions for the partition functions of the finite N *D*-instanton matrix model in $d=4$ and 6 dimensions that generalize our previous expression for the case $d=10$. These results are consistent with the fact that the Witten index of the *T*-dual *D*-particle process should only be nonvanishing for $d=10$. [S0556-2821(98)04616-5]

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I. INTRODUCTION

When Yang-Mills theory with gauge group G is reduced to zero space-time dimensions it is a theory of vector potentials A_μ (where $\mu=0,1,\ldots,d-1$ labels d Euclidean dimensions) that are constant matrices and the action is simply proportional to $\mathrm{tr}[A_\mu,A_\nu]^2$. This system is of relevance in calculating the zero-mode contribution to Yang-Mills theory in a box [1]. When G is SU(N) (so that A_μ is a Hermitian traceless $N\times N$ matrix) and in the limit $N\to\infty$ this zero-dimensional model was shown by Eguchi and Kawai to encode the information in d-dimensional Yang-Mills theory [2], at least in the quenched version of the model.

Adding fermions in a supersymmetric manner dramatically changes the nature of the model. In supersymmetric SU(N) Yang-Mills theory, which exists in $d=3,4,6,10$, the fermions ψ^a are space-time spinors ($a=1,\ldots,2^{d/2-1}$ for $d\neq 3$ and $a=1,2$ for $d=3$) and are also $N\times N$ matrices.[1] In these cases the partition function can be written as

$$Z_N^{(d)}=\frac{1}{\mathrm{Vol}(\mathrm{SU}(N))}\int DAD\psi e^{-1/g S_{\mathrm{YM}}[A,\psi]}, \qquad (1.1)$$

where S_{YM} is the supersymmetric Yang-Mills action in d dimensions reduced to a point,

$$S_{\mathrm{YM}}=\frac{1}{4}\,\mathrm{tr}([A_\mu,A_\nu]^2)+\frac{i}{2}\,\mathrm{tr}(\bar{\psi}\Gamma_\mu[A^\mu,\psi]), \qquad (1.2)$$

and g is the string coupling constant. The large-N limit of the $d=10$ model has been used to define a version of the matrix theory [3,4] that is a candidate model for a nonperturbative description of type IIB superstring theory. An important effect of the supersymmetry is to ensure that there are exactly flat directions along which the eigenvalues of A_μ^I feel no potential [where I labels elements of the Cartan subalgebra of SU(N)]. This can be seen already in the $N=2$ (2 *D*-instanton) system where the behavior of the ‘‘quenched’’ system, in which the instantons are separated by a fixed distance L, is qualitatively different for the supersymmetric cases [5].

The partition function $Z_N^{(d)}$ is also of interest in a separate context. It may be identified with the bulk contribution to the Witten index for the system of N interacting D particles in the T-dual type IIA string theory. Indeed, it was in the context of the Witten index that the partition function was evaluated explicitly for the case $N=2$ in [6,7] where it was shown that $Z_2^{(3)}=0$, $Z_2^{(4)}=Z_2^{(6)}=1/4$, and $Z_2^{(10)}=5/4$. These values are in accord with the lore concerning the presence of D-particle threshold bound states in $d=10$ Yang-Mills matrix quantum mechanics and the absence of bound states in the lower dimensional cases. This lore is based on a variety of duality arguments. D particles in $d=10$ are supposed to be identified with Kaluza-Klein modes of 11-dimensional supergravity and the presence of threshold bound states corresponds to the multiply charged modes. One signal for these states is that the Witten index for N D particles should equal one. The $d=6$ supersymmetric Yang-Mills matrix model can be obtained as a limit of type IIA string theory compactified on $K3$. This is a scaling limit in which a two-cycle in the $K3$ vanishes. A $D2$ brane wrapped around the cycle is massless at the degeneration point and is interpreted as a Yang-Mills gauge particle in the dual heterotic picture [8]. Since there is a single Yang-Mills state it is important that multiply wrapped $D2$ branes do not give rise to new threshold bound states and the Witten index should vanish. A similar argument applies to the four-dimensional theory obtained by compactifying type IIB string theory on a Calabi-Yau space in the limit in which a three-cycle is degenerating. As shown by Strominger [9] the singularity in the classical vector moduli space is resolved by the presence of new massless states associated with a $D3$ brane wrapped once around the cycle. Again, multiple wrappings must not give extra normalizable states if the mechanism for resolving the singularity is to work. The $d=3$ partition function $Z_N^{(3)}$ is believed to

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[1] We will define the fermions with the Minkowski signature for space-time before passing to a Euclidean signature.

vanish [7,6,10] so this case is qualitatively different from the higher d cases. The relevant Euclidean brane configuration would be the Euclidean $D2$ brane wrapping a three-cycle in a Calabi-Yau fourfold. However, such cycles do not preserve half the supersymmetries [11].

In [12] we demonstrated how the $d=10$ zero-dimensional matrix model partition function for arbitrary N, $Z_N^{(10)}$, could be extracted from the expressions for certain protected terms in the type IIB effective action (such as the R^4 term). We also showed how this is compatible with a nonvanishing Witten index. This discussion will be reviewed and extended in Sec. II. The $d=6$ case will be considered in Sec. III where the matrix model of relevance close to a degeneration point at which a single two-cycle in $K3$ shrinks to zero volume will be arrived at by considering a scaling limit of the threshold corrections derived in [13]. In Sec. IV this will be further extended to the $d=4$ case where the matrix model is associated with the degeneration of three-cycles in Calabi-Yau spaces of [9].

In all these cases we are interested in extracting the leading instanton contributions to the various terms in the effective action. These terms can be expressed in the string frame in the form

$$\int d^dx\sqrt{g}F_{\mathcal{P}}^{(d)}(\rho,\overline{\rho})\mathcal{P}(\Psi)+\text{c.c.}, \tag{1.3}$$

where $\mathcal{P}(\Psi)$ is a combination of fields (such as R^4 and related terms in the $d=10$ case) and $\rho=\chi+ie^{-\phi}$ is a complex combination of Ramond$\otimes$Ramond (R$\otimes$R) and Neveu-Schwarz$\otimes$Neveu-Schwarz (NS$\otimes$NS) scalar fields. The particular moduli that are identified as χ and ϕ depend on the context. For large $e^{-\phi}$ (weak coupling) the function $F_{\mathcal{P}}^{(d)}(\rho,\overline{\rho})$ can be written as an expansion that has the generic form

$$F_{\mathcal{P}}^{(d)}=\text{pert.}+\sum_{N\neq 0}\mathcal{G}_{N,\mathcal{P}}^{(d)}(\rho,\overline{\rho}), \tag{1.4}$$

where pert. denotes a finite number of perturbative contributions and $\mathcal{G}_{N,\mathcal{P}}^{(d)}$ contains N-instanton effects. For the leading order in the weak coupling expansion this has the form

$$\begin{aligned}\mathcal{G}_{N,\mathcal{P}}^{(d)}&\sim cZ_N^{(d)}(S_N)^{a_d+p}e^{-2\pi(S_N+iN_\chi)}(1+o(1/Ne^{-\phi}))\\&\equiv\mathcal{Z}_N^{(d)}S_N^p,\end{aligned} \tag{1.5}$$

where the constant a_d depends only on the dimension d while p is the number of fields in the interaction $\mathcal{P}$ in linearized approximation and S_N is the N-instanton action $S_N=Ne^{-\phi}$. The quantity $Z_N^{(d)}$, is identified with the partition function defined earlier in terms of the zero-dimensional reduction of super Yang-Mills theory. The measure $\mathcal{Z}_N^{(d)}=(S_N)^{a_d}e^{-2\pi(S_N+iN_\chi)}$, defined by Eq. (1.5), is independent of the interaction term being considered. The general structure of these interactions can be obtained by considering perturbative string contributions around a D-instanton background. This requires a sum over world sheets that include configurations of disconnected disks with Dirichlet boundary conditions, as described in [14] for the $d=10$ case (and in [15,16] for the bosonic string). The lowest nontrivial perturbative contributions of this kind will be evaluated for the $d=4$ case in Sec. IV A in order to confirm the general structure of these corrections, although perturbation theory alone cannot verify the detailed form of $Z_N^{(d)}$.

We will discover that

$$Z_N^{(10)}=\sum_{m|N}\frac{1}{m^2},\quad Z_N^{(6)}=Z_N^{(4)}=\frac{1}{N^2}. \tag{1.6}$$

The $d=10$ result was given in [12] and agrees with the expectation that the Witten index should equal one. The $d=6$ and $d=4$ results agree with the expectation that the Witten index vanishes in these cases as well as with the results of [17]. The relation of these results to the arguments in [18] is obscure.

II. $d=10$ AND M-THEORY-TYPE IIB DUALITY

An important ingredient of the web of nonperturbative dualities is the correspondence between M theory on T^2 and type II string theory on S^1. The original arguments for this were motivated by the form of the leading terms in the low energy effective actions for these theories [19–21]. This generalizes to the richer structure of the next terms in the low energy expansion that include, for example, the R^4 terms and other terms of the same dimension. We will review the form of these terms and how they are related to the Witten index for the $d=10$ D-particle system in this section.

The connection with the Witten index was made in [12] where we suggested how the analysis of the two D-particle system in [6] and [7] should generalize to an arbitrary number of D particles in the case of $d=10$. For any value of d the index has the form

$$I^{(N)}=\lim_{\beta\to\infty}\text{tr}(-1)^Fe^{-\beta H_N}\equiv I_{\text{bulk}}^{(N)}+\delta I^{(N)}, \tag{2.1}$$

where H_N is the N D-particle Hamiltonian. $I_{\text{bulk}}^{(N)}$ is the bulk contribution to the index and is identical to the zero-dimensional partition function

$$I_{\text{bulk}}^{(N)}=\lim_{\beta\to 0}\text{tr}(-1)^Fe^{-\beta H_N}=Z_N^{(d)}, \tag{2.2}$$

and $\delta I^{(N)}$ is a boundary contribution given by

$$\delta I^{(N)}=\int_0^\infty d\beta\,\frac{d}{d\beta}\,\text{tr}(-1)^Fe^{-\beta H_N}. \tag{2.3}$$

The boundary term was found to arise in the $N=2$ case from the region of moduli space in which the two particles are separated and noninteracting [6,7]. This region is described by quantum mechanics of two free particles on the orbifold space R^{d-1}/S_2. In [12] it was assumed that, in the $d=10$ case, for general N the boundary term comes from the obvious generalization of this region to the regions of moduli space in which m (>1) D particles of charge k are separated, where $N=km$. A key assumption is that the separated charge-k bound states also behave as free particles. This region is then described by quantum mechanics on $R^{9(m-1)}/S_m$ and summing over the all values of m leads to the expression in this case of

$$\delta I^{(N)}=-\sum_{\substack{m|N\\m>1}}\frac{1}{m^2}. \tag{2.4}$$

The Witten index follows once $I^{(N)}_{\rm bulk}$ is evaluated.

By making use of Eq. (2.2) the expression for $I^{(N)}_{\rm bulk}$ can be identified with the coefficient of the N-instanton contribution to the expansion of certain protected terms in the type IIB effective action. These terms include [14,22,23]

$$\int d^{10}x\sqrt{g}e^{-\phi/2}[f_4(\rho,\bar\rho)R^4+f_{16}(\rho,\bar\rho)\lambda^{16}+\cdots], \tag{2.5}$$

where R^4 denotes a specific contraction of four Riemann curvatures, λ^{16} is a specific 16-fermion term (where λ is the spin-1/2 Weyl fermion of the type IIB theory), and the terms indicated by dots are other terms of the same dimension that are related by supersymmetry [23,24]. Such terms include $\psi^2\psi^{*2},f_4G^2G^{*2},f_8G^8$ and many others, where ψ_μ is the Weyl gravitino and G is a complex combination of the Ramond-Ramond (R⊗R) and the Neveu-Schwarz–Neveu-Schwarz (NS⊗NS) antisymmetric tensor field strengths.[2] The functions f_4, f_{16}, and the other coefficient functions are nonholomorphic modular forms that depend on the complex scalar field

$$\rho=c^{(0)}+ie^{-\phi}, \tag{2.6}$$

where $c^{(0)}$ is the R⊗R scalar and ϕ is the dilaton.[3] These coefficients are generalized (nonholomorphic) Eisenstein series and are given by

$$f_p=\rho_2^{3/2}\sum_{m,n\neq 0}(m+n\bar\rho)^{p-11/2}(m+n\rho)^{-p+5/2}, \tag{2.7}$$

which transforms under modular transformations as a form of holomorphic weight $(p-4)$ and antiholomorphic weight $-p+4$ which we will write as

$$\text{Weight}\ \ f_p=(p-4,-p+4). \tag{2.8}$$

These functions are related to each other by the action of a covariant derivative,

$$f_p=2^p\frac{\Gamma(-5/2)}{\Gamma(p-5/2)}(\rho_2\mathcal{D})^pf_0, \tag{2.9}$$

where

$$\mathcal{D}=\frac{i\partial}{\partial\rho}+\frac{d}{2\rho_2}, \tag{2.10}$$

acting on a $(d,\bar d)$ form converts it into a $(d+2,\bar d)$ form.

The expansion of the coefficient functions for small coupling, $e^\phi\to 0$, gives perturbative tree-level and one-loop contributions together with an infinite number of D-instanton (and anti–D-instanton) terms. The absence of higher order perturbative corrections is presumably related to the fact that the terms in Eq. (2.5) are given by integrals over half the on-shell superspace. An indirect argument for such a nonrenormalization theorem has been advanced in [35]. The expansion of f_p can be written in the form (1.4):

$$e^{-\phi/2}f_p=2\zeta(3)e^{-2\phi}+\frac{2\pi^2}{3}c_p+\sum_{N=1}^{\infty}\mathcal{G}^{(10)}_{N,p}, \tag{2.11}$$

where [33]

$$c_p=(-1)^p\frac{\pi}{4}\frac{1}{\Gamma(-5/2+p)\Gamma(11/2-p)}. \tag{2.12}$$

The first two terms in Eq. (2.11) have the interpretation of the tree-level and one-loop string terms while the instanton and anti-instanton terms are contained in the asymptotic series:

$$\mathcal{G}^{(10)}_{N,p}=(8\pi)^{1/2}\left(\sum_{N|m}\frac{1}{m^2}\right)(2\pi N\rho_2)^{1/2}\times\left[\sum_{r=4-p}^{\infty}\frac{c_{-p,r}}{(2\pi N\rho_2)^r}e^{2\pi iN\rho}+\sum_{r=p-4}^{\infty}\frac{c_{p,r}}{(2\pi N\rho_2)^r}e^{-2\pi iN\bar\rho}\right], \tag{2.13}$$

where

$$c_{p,r}=\frac{(-1)^p}{2^r(r-p+4)!}\frac{\Gamma(3/2)}{\Gamma(p-5/2)}\frac{\Gamma(r-1/2)}{\Gamma(-r-1/2)}. \tag{2.14}$$

[2] For related work on D-instanton effects in toroidally compactified type II theories see [25–27] and in type I theories see [28–30]. $SL(2,Z)$-invariant expressions for higher dimensional terms in the type IIB effective action have also been proposed in [31–34,24].

[3] In earlier papers [14,23] the function f_4 was denoted f—a more uniform notation is adopted here.

The expression (2.13) is somewhat formal since the series multiplying each (anti)instanton contribution are asymptotic expansions of Bessel functions for large arguments.

From Eq. (2.13) we see that the leading contribution to the N D-instanton contribution in $\mathcal{G}_{N,p}^{(10)}(\rho,\overline{\rho})$ is

$$\mathcal{G}_{N,p}^{(10)} \sim Z_N^{(10)}(2\pi S_N)^{-7/2+p} e^{-2\pi(S_N+iNc^{(0)})}(1+o(e^{\phi})), \tag{2.15}$$

where the N D-instanton action is given by $S_N = Ne^{-\phi}$. This expression is of the form (1.5) with $a_{10} = -7/2$, χ identified with $c^{(0)}$ and with

$$Z_N^{(10)} = \sum_{m|N} \frac{1}{m^2}, \tag{2.16}$$

which has been normalized so that $Z_1^{(10)} = 1$.

The power $a_{10} = -7/2$ in Eq. (1.5) arises from the combination of ten bosonic zero modes (each contributing $S_N^{1/4}$) and 16 fermionic zero modes (each contributing $S_N^{-3/8}$). The number p is the number of external fields in the linearized approximation to the interaction term ($p=4$ for R^4, $p=16$ for λ^{16}, etc.).

We have thus determined the factor that is to be identified with the partition function of the SU(N) zero-dimensional matrix mode. Combining Eq. (2.16) with Eqs. (2.2) and (2.4) gives the result for the Witten index (2.1) of the $d=10$ case:

$$I^{(N)} = 1, \tag{2.17}$$

which indicates the presence of at least one bound state for each value of N.

III. $d=6$ AND HETEROTIC-TYPE II DUALITY

In this section we wish to consider type IIA compactified to six dimensions on $K3$, which is equivalent, via strong coupling duality, to the heterotic string on T^4. This theory has two eight component supercharges. At special points in the heterotic moduli space $\mathcal{M}_{4,20}$ additional massless states appear, leading to a perturbative enhancement of the gauge symmetry. On the type IIA side these are associated with $D2$ branes wrapping a vanishing S^2 in the $K3$. The absence of bound states of multiply wrapped $D2$ branes is demanded by the absence of an infinite tower of such massless gauge states on the heterotic side. A T-duality transformation (in the Euclidean time direction), maps the $D2$ brane of type IIA to the Euclidean world sheet of a D string in type IIB. This is a D instanton from the six-dimensional point of view.

Since the S^2 is chosen to be a supersymmetric cycle such an instanton preserves half of the supersymmetries [36]. The eight broken supersymmetries generate fermionic collective coordinates which have to be soaked up by external sources in order to give nonzero correlation functions in an instanton background, thereby generating new interaction vertices. The simplest such term will be an eight-fermion term. Among the other terms related to this by supersymmetry is the four-derivative term $\partial_\mu\phi\partial_\nu\phi\partial^\mu\partial^\nu\phi$. In the background of N D instantons such vertices are weighted with a factor $\exp(-S_N)$ where the N D instanton action is given by

$$S_N = Ne^{-\phi_6}\,\mathrm{Vol}(S^2), \tag{3.1}$$

where ϕ_6 is the $d=6$ dilaton. We will obtain the expression for $Z_N^{(6)}$ from the form of these four-derivative interactions in the effective action.

This is seen by considering one-loop threshold corrections to the heterotic string on $T^6 = T^4 \times T^2$ [13]. One of these is of the form $F_1 R^2$ term where F_1 is a function of the vector multiplet moduli while the other is $\tilde{F}_1 \partial_\mu\phi\partial_\nu\phi\partial^\mu\partial^\nu\phi$ where $\tilde{F}_1$ depends on the hypermultiplet moduli only. Since the heterotic dilaton lies in a vector multiplet the heterotic one-loop calculation of the threshold function $\tilde{F}_1$ is exact. In this manner the form of $\tilde{F}_1$ may be determined in type IIA on $K3\times T^2$, which may then be related by T duality in one of the T^2 directions to type IIB. This duality, which exchanges the Kähler modulus and complex modulus of T^2, exchanges a two-brane wrapped on a two-cycle for the world sheet of a D string wrapped on the same cycle. The six-dimensional four-derivative terms can then be found by taking the large volume limit of the type IIB T^2.

Thus, following [13], we may write the six-dimensional one-loop result for $\tilde{F}_1$ in terms of type IIB variables and extract the various perturbative and nonperturbative contributions by expanding in the appropriate couplings. The expression for $\tilde{F}_1$ is given in Eq. (6.2) of [13]:

$$\tilde{F}_1 = 8\pi + 2e^{-\phi_6} \sum_{N\neq 0,\, q^i\neq 0} \frac{1}{|m|} C\left(\frac{q^t L q}{2}\right)(q^t(M+L)q)^{1/2} K_1\left[2\pi|N|\left(\frac{q^t(M+L)q}{2}\right)^{1/2} e^{-\phi_6}\right] e^{2\pi i N Y_i q^i}. \tag{3.2}$$

The matrix M parametrizes the moduli space $O(20,4,Z)/O(4,20,R)/O(4,R)\times O(20,R)$, L is the metric on the signature (20,4) Narain lattice and q^i are integer charges that parametrize the lattice momenta (p_l and p_r) that satisfy

$$p_l^2 = \tfrac{1}{2} q^t(M+L)q, \quad p_r^2 = \tfrac{1}{2} q^t(M-L)q. \tag{3.3}$$

The level matching and mass-shell conditions for perturbative heterotic states are

$$\tfrac{1}{2}(p_l^2 - p_r^2) = N_l - N_r + 1, \quad m^2 = \tfrac{1}{2}(p_l^2 + p_r^2) + N_l + N_r - 1. \tag{3.4}$$

The degeneracy factor $C(k)$ in Eq. (3.2) is defined by $\eta^{-24}(\tau) = \Sigma_{k>-2} C(k) e^{-k\tau}$ (η is the Dedekind function) and ϕ_6 is the six-dimensional type IIB dilaton. The Wilson lines Y_i of the heterotic string correspond to R⊗R fields dimensionally reduced on $K3$ on the type II side.

In [13] the expression (3.2) was used to determine the nonperturbative effects in the decompactified limit in which the volume of the $K3$ is infinite, which confirms the form of the $d=10$ D-instanton terms described in Sec. II. Here, we wish to use Eq. (3.2) to extract the $d=6$ D-instanton corrections coming from Euclidean D branes wrapping the two-cycles of $K3$. Using the mass-shell condition it is easy to see from the heterotic side that the extra state with vanishing mass has a charge satisfying $q^t Lq/2 = -1$ and a mass given by

$$\mu^2 = \frac{q^t(M+L)q}{2}. \tag{3.5}$$

To isolate the instanton effects on the type IIB side we tune the moduli M in such a way that $\mu \to 0$ for a specific charge vector q_i, such that $\mu e^{-\phi_6}$ is fixed. The degeneracy of this state is $C=1$. Expanding the Bessel function K_1 for large $\mu e^{-\phi_6}$ gives the leading nonperturbative contribution:

$$\tilde{F}_1 \sim \sum_{N\neq 0} e^{-\phi_6/2} \frac{\mu^{1/2}}{|N|^{3/2}} e^{-2\pi|N|\mu e^{-\phi_6}} e^{2\pi i N \int_{s^2} c^{(2)}} + \text{c.c.}$$

$$= \sum_{N\neq 0} \frac{1}{N^2} (S_N)^{1/2} e^{-2\pi S_N + 2\pi i N \int_{s^2} c^{(2)}} + \text{c.c.}, \tag{3.6}$$

where we have kept only the first term in an asymptotic series of perturbative fluctuations around the D-instanton background, $c^{(2)}$ is the two-form R⊗R potential that couples to the wrapped D-string world volume, and we have used $\mu = \text{Vol}(S^2)/\alpha'$ so that $S_N = |N|\mu e^{-\phi_6}$ is the instanton action. This expression again has the form (1.5) with $a_6 = -3/2$ and $p=2$, where χ is identified with $\int c^{(2)}$. With this identification we determine

$$Z_N^{(6)} = \frac{1}{N^2}, \tag{3.7}$$

a result which was also derived in [17] using apparently different arguments.

This result implies that the bulk term in the Witten vertex is $I_{\text{bulk}}^{(N)} = 1/N^2$. In the absence of multi–D-particle threshold bound states (multiply wrapped $D2$ branes) the boundary contribution to the index is that of N free particles, namely, $\delta I^{(N)} = -1/N^2$. This is consistent with the expectation that the Witten index $I_{\text{bulk}}^{(N)} + \delta I^{(N)}$ should vanish in the $d=6$ case.

IV. $d=4$ AND THE CONIFOLD

We turn now to consider the type II string theories compactified on a Calabi-Yau threefold near a conifold singularity. In the simplest case a nontrivial three cycle γ with period

$$z = \int_\gamma \Omega \tag{4.1}$$

vanishes in the conifold limit. As pointed out by Strominger [9] the singularity in the vector multiplet moduli space of the classical type IIB theory is interpreted in the low energy quantum theory as the one-loop effect of a light hypermultiplet produced by a $D3$ brane wrapping the cycle γ. In contrast to the vector multiplet moduli space the hypermultiplet moduli space can receive both perturbative and nonperturbative corrections. It was suggested in [36] that for type IIA on the same Calabi-Yau space Euclidean $D2$ branes wrapping γ lead to large instanton effects which smooth out the classical singularity of the hypermultiplet moduli space. Following field theoretical arguments given in [37] a corrected metric on the moduli space near the conifold was derived in [38]. The metric was determined in the limit $z\to 0$ keeping $|z|/\lambda$ fixed (where λ is the string coupling), in which the details of how the conifold is embedded in the Calabi-Yau do not play any role (neither does the fact that the hypermultiplets parametrize a quaternionic instead of a hyper-Kähler geometry). The expression for the metric in [38] is given (in the string frame) by

$$ds^2 = V^{-1}\left(dt - A_x dx - \frac{1}{\lambda} A_z d\bar{z} - \frac{1}{\lambda} A_{\bar{z}} dz\right)^2 + V\left(dx^2 + \frac{1}{\lambda^2} dz d\bar{z}\right). \tag{4.2}$$

The four scalars in the hypermultiplet comprise the complex field z associated with the complex structure deformation parametrizing the conifold limit and x,t which are the reduced R⊗R three forms corresponding to the elements of the cohomology associated with the cycle γ and the dual cycle, respectively. The scalar potential V in Eq. (4.2) is given by[4]

$$V = \frac{1}{4\pi} \ln\left(\frac{\Lambda^2}{|z|^2}\right) + \frac{1}{\pi} \sum_{N>0} \cos(2\pi N x) K_0(2\pi N|z|/\lambda), \tag{4.3}$$

and the vector potential A is determined by $\nabla V = \nabla \times A$ to be,

$$A_x = -\frac{1}{2\pi}\theta,$$

$$A_z = A_{\bar{z}} = \frac{1}{\pi} \sum_{N>0} \sin(2\pi N x) K_1(2\pi|Nz|/\lambda). \tag{4.4}$$

The instanton terms again become apparent by expanding the Bessel function K_0 for large values of $|z|/\lambda$,

$$V \sim \frac{1}{4\pi} \ln\left(\frac{\Lambda^2 e^{2\Phi}}{\lambda^2}\right) + \sqrt{\frac{2}{\pi}} \sum_{N>0} (2\pi N e^{-\Phi})^{-1/2} e^{-2\pi N e^{-\Phi}} \times \cos(2\pi N x)(1 + o(1/Ne^{-\Phi})), \tag{4.5}$$

where we have set $z = \lambda e^{-\Phi + i\theta}$ so that $|z|/\lambda = e^{-\Phi}$ and

$$dz d\bar{z} = \lambda^2 e^{-2\Phi}(d\Phi + i d\theta)(d\Phi - i d\theta). \tag{4.6}$$

From this it follows that the nonperturbative contribution to the $d\Phi d\Phi$ component of the metric has the form

$$\sum_{N\neq 0} \mathcal{G}^{(4)}_{N,\Phi\Phi} d\Phi d\Phi, \tag{4.7}$$

[4]In order to avoid notational confusion we are using the symbol Λ instead of μ to represent the renormalization scale.

where $\mathcal{G}^{(4)}_{N,\Phi\Phi}$ has a leading N-instanton contribution of the form (4.5),

$$\mathcal{G}^{(4)}_{N,\Phi\Phi}\sim\frac{1}{2\pi}\frac{1}{N^2}S_N^{3/2}e^{2\pi iNx}e^{-2\pi S_N}+\text{c.c.},\tag{4.8}$$

and where the action for a charge N instanton is given by

$$S_N=|N|e^{-\Phi}.\tag{4.9}$$

In writing Eq. (4.8) we have kept only the leading contribution to the N-instanton term, which does not get contributions from the expansion of the expression for A_z in Eq. (4.4). The expression (4.8) is again of the form (1.5) with χ identified with x, $p=2$, $a_4=-1/2$ and with

$$Z_N^{(4)}=\frac{1}{N^2}.\tag{4.10}$$

As with the $d=6$ case this expression is consistent with the absence of D-particle bound states (and agrees with the result in [17]).

In identifying the measure (4.8) it was important that we used the metric for the fluctuations of Φ rather than of the field z. These metrics differ by a factor of $\lambda^2e^{-2\Phi}$. A way to check that this choice of normalization is appropriate for extracting the form of $Z_N^{(4)}$ is to emulate the way in which the instanton measure can be evaluated in field theory by considering the contributions of perturbative fluctuations around the instanton background to correlation functions.

A. Perturbative fluctuations around a stringy *D* instanton

The appearance of D-instanton induced terms in the effective action can be seen in string perturbation theory around an instanton background following the same kind of arguments made in [14] for the ten-dimensional theory. For the lowest order in a perturbative expansion the world sheet consists of a number of disconnected disks with closed-string vertex operator insertions and with D-instanton boundary conditions which can be implemented by constructing an appropriate boundary state. The D instanton preserves half the space-time supersymmetry so that a combination of the left-moving and right-moving space-time supersymmetry charges annihilate the boundary state. Applying n from the broken supersymmetries to the boundary state generates a boundary state, $|B\rangle_n$, where n denotes the number of fermionic zero modes, which correspond to fermionic open string ground states attached to the boundary. Nonvanishing correlation functions arise when a sufficient number of these fermionic modes are integrated—sixteen for the type IIB theory in $d=10$ considered in [14] (and Sec. II), eight in the $d=6$ case in Sec. III and four in the $d=4$ case in Sec. IV.

Here we will only consider the $d=4$ case where the instantons are Euclidean $D2$ branes wrapped on a three-cycle of a Calabi-Yau threefold. The implementation of the boundary conditions in this case, as well as the construction and properties of the associated boundary states were discussed in detail in [39]. Type IIA compactified on a Calabi-Yau manifold has eight real supersymmetries and a Euclidean $D2$ brane wrapped on a supersymmetric three-cycle preserves four of these. Hence there are four fermionic zero modes and this leads, for example, to a ''t Hooft'' four-fermion interaction vertex [36]. The leading contribution to this interaction comes from a world sheet consisting of four disconnected disks with a fermion vertex operator attached to the interior of each and one fermionic zero mode (open string) attached to each boundary. This four-fermion vertex is related by supersymmetry to the metric for the hypermultiplets that we have been considering. The leading instanton contribution to this metric comes from a world sheet consisting of two disconnected disks with a vertex for a hypermultiplet modulus inserted to the interior of each and two fermionic zero modes attached to each boundary.

The vertex for the hypermultiplet modulus Φ^i is given by

$$V^i_\Phi=e^{-\phi}e^{-\bar{\phi}}\Phi^i_{q,\bar{q}}e^{ikX},\tag{4.11}$$

where q and $\bar{q}$ denote the left and rightmoving U(1) charges of the field Φ^i of the internal $c=9$ $N=2$ superconformal field theory, which is associated with the compactification on the Calabi-Yau manifold. For type IIA compactified on a Calabi-Yau the hypermultiplets parametrize the complex structure moduli space and the scalars in the NS$\otimes$NS sector are associated with elements of the cohomology $H^{2,1}$ and $H^{1,2}$ which are given by (c,c) primary fields $\Phi^i_{1,1}$ and (a,a) fields $\Phi^i_{-1,-1}$, respectively. The R$\otimes$R vertices are related to these by spectral flow.

In [39] it was shown that there are two possible boundary conditions called A and B [connected to the two topological twists of $N=2$ super conformal field theory (SCFT) [40]] which correspond to branes wrapping middle and even dimensional cycles, respectively. Hence the A boundary conditions are relevant in our case and the supersymmetry charges are given in the $-1/2$ picture by

$$Q^a_{-1/2\pm}=e^{-1/2\phi}S^ae^{-i(\sqrt{3}/2)H}\pm ie^{-1/2\bar{\phi}}\bar{S}^ae^{i(\sqrt{3}/2)\bar{H}},$$
$$Q^{\dot{a}}_{-1/2\pm}=e^{-1/2\phi}S^{\dot{a}}e^{i(\sqrt{3}/2)H}\pm ie^{-1/2\bar{\phi}}\bar{S}^{\dot{a}}e^{-i(\sqrt{3}/2)\bar{H}},\tag{4.12}$$

and in the $+1/2$ picture by

$$Q^a_{1/2\pm}=e^{1/2\phi}(\gamma_\mu S)^a\partial X^\mu e^{-i(\sqrt{3}/2)H}\pm ie^{1/2\bar{\phi}}(\gamma_\mu\bar{S})^a\bar{\partial}X^\mu e^{i(\sqrt{3}/2)\bar{H}},$$
$$Q^{\dot{a}}_{1/2\pm}=e^{1/2\phi}(\gamma_\mu S)^{\dot{a}}\partial X^\mu e^{-i(\sqrt{3}/2)H}\pm ie^{1/2\bar{\phi}}(\gamma_\mu\bar{S})^{\dot{a}}\bar{\partial}X^\mu e^{i(\sqrt{3}/2)\bar{H}}.\tag{4.13}$$

Here the unbarred fields denote leftmovers and the barred fields denote rightmovers, ϕ denotes the bosonized superghost, S^a and $S^{\dot{a}}$ are SO(4) spin fields of opposite chirality, and H is the free boson associated with the U(1) current of the internal $c=9$ SCFT. The A boundary condition enforces $Q_+|B\rangle=0$ and Q_- are the vertex operators for the fermionic collective coordinates of the D instanton. The disk amplitude is then given by inserting one scalar vertex and two vertices for supercharges of the broken supersymmetries (such amplitudes were evaluated in the ten-dimensional case in [14,41]):

$$\phi_i \epsilon_1^a \epsilon_2^{\dot{a}} \left\langle c\bar{c} e^{-\phi} e^{-\bar{\phi}} \Phi^i_{q,\bar{q}} e^{ikX}(z) c Q^a_-(x_1) \int dx_2 Q^{\dot{a}}_-(x_2) \right\rangle$$
$$= \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu \phi_i \frac{N}{\lambda} \langle \Phi^i \rangle, \qquad (4.14)$$

where ϕ^i is the (on-shell) wave function for the field Φ^i. One supersymmetry charge is in the 1/2 and one in the $-1/2$ picture in order for the total ghost number on the disk to add up to -2. The notation $\langle \Phi^i \rangle$ denotes the expectation value on the disk of the R⊗R field $\Phi^i_{-1/2,-1/2}$ which is associated to $\Phi_{1,1}$ by spectral flow and is the same as the topological amplitude derived in [39]. There it was shown that $\langle \Phi^i \rangle$ is independent of the Kähler moduli and is given by

$$\langle \Phi^i \rangle = \int_\gamma \omega^i = D_i \int_\gamma \Omega, \qquad (4.15)$$

where D denotes the covariant derivative on the vacuum line bundle over the moduli space of the $N=2$ SCFT [38].

The lowest order term in the correction to the metric comes from a configuration of two disks. One of these has a vertex operator for the modulus ϕ^i attached and the other has the vertex operator for the complex conjugate $\phi^{\bar{j}}$. After integration over the fermionic zero modes and summation over the instanton sectors the expression reduces to

$$\sum_{N\neq 0} \int d^4\epsilon \langle V_{\phi^i} \epsilon Q \epsilon Q \rangle \langle V_{\phi^{\bar{j}}} \epsilon Q \epsilon Q \rangle \mathcal{Z}_N^{(4)}$$
$$= \partial_\mu \phi^i \partial^\mu \phi^{\bar{j}} \sum_{N\neq 0} \frac{N^2}{\lambda^2} \mathcal{Z}_N^{(4)} D_i \int_\gamma \Omega D_{\bar{j}} \int_\gamma \bar{\Omega}, \qquad (4.16)$$

where $Z_N^{(4)}$ is to be identified with the N-instanton measure—it is a factor that does not depend on the particular process being considered.

We now specialize for the case where the vertices $\phi_i, \phi_{\bar{j}}$ are the moduli $z, \bar{z}$, respectively. In the conifold limit $D_z \int_\gamma \Omega = 1$ and it follows that the instanton correction to the metric is given by

$$\sum_{N\neq 0} \frac{N^2}{\lambda^2} \mathcal{Z}_N^{(4)} dz d\bar{z} = \sum_{N\neq 0} (N e^{-\Phi})^2 \mathcal{Z}_N^{(4)} (d\Phi d\Phi + d\theta d\theta)$$
$$= \sum_{N\neq 0} \mathcal{G}^{(4)}_{N,\Phi\Phi} (d\Phi d\Phi + d\theta d\theta). \qquad (4.17)$$

In order for Eq. (4.17) to agree with Eq. (4.7), $\mathcal{G}^{(4)}_{N,\Phi\Phi}$ must be identified with Eq. (4.8). In the analysis of the instanton induced corrections of the metric $Z_N^{(4)}$ is interpreted as the instanton measure which is independent of the process considered. On the other hand the disk amplitudes (4.14) depend on the fluctuating fields. In the case of the fluctuations of the field z the disk amplitude (4.14) is proportional to N/λ which is not equal to the instanton action S_N. In order to bring Eq. (4.17) into the general form (1.5) with $a_4 = -1/2$ and $p=2$ it is necessary to consider the fluctuations of the fields ϕ and θ instead of $z, \bar{z}$ which can be accomplished by the change of variables (4.6).

The leading instanton correction for to the metric for the R⊗R scalar x is given by $V dx dx$ and is also reproduced by the two-disk process. The R⊗R fields have a different dependence on the string coupling (the one-point function is $\langle V_x \rangle = N$ in this case) so it is important to remember that the canonically normalized perturbative field is $\hat{x} = x e^{-\Phi}$. The metric is then of the form

$$\frac{1}{N^2} (S_N)^2 V d\hat{x} d\hat{x}, \qquad (4.18)$$

which may be expressed as Eq. (1.5) with $a_4 = -1/2$ and $p=2$.

All the other corrections in Eq. (4.2) are subleading in the coupling constant expansion and should correspond to more complicated diagrams in the instanton background which will not be discussed here. It would be interesting to derive the form of the metric (4.2) from the duality of type II on Calabi-Yau spaces and the heterotic string on $K3 \times T^2$. As in the discussion of threshold corrections in the $d=6$ case, since the heterotic dilaton lies in a vector multiplet the non-perturbative effects on the type II side must be reproduced by tree-level effects on the heterotic side (taking into account all orders in α'). Unfortunately, relatively little is known about the structure of such hypermultiplet moduli spaces from the heterotic point of view (for a recent review see [42]).

V. DISCUSSION

We have considered the contributions of D instantons to a variety of protected interactions in $d=4$, 6, and 10 dimensions. There appear to be no examples of such instanton effects when $d=3$, which is consistent with the expected vanishing of $Z_N^{(3)}$. The interaction terms considered are ones that are protected by supersymmetry and in which only multiply charged single D instantons contribute since the contributions of separated multi-instantons carry extra fermionic zero modes and therefore vanish. We are thus able to isolate the instanton partition function $Z_N^{(d)}$, which also only gets contributions from single instantons.

In all the cases considered in this paper the basic structure of the N-instanton calculation in d dimensions follows from a one-loop calculation $R^{d-1} \times S^1$ with a circulating D particle of the appropriate type. Using T duality the charge-N D instanton corresponds to the contribution of an N fold winding of the Euclidean world line of the D particle around the S^1. An expression of the form (1.5) follows in a simple manner in each case, as is summarized in the Appendix. It is important to note that in all cases there is an infinite tower of multiply charged D instantons.

The correspondence, via Euclidean T duality, between D instantons and D particles[5] makes the connection with the

[5]Some aspects of this relation were discussed in [43,44].

Witten index clear. In $d=4$ and $d=6$ the D particle is the unique normalizable state—described by a $D3$ brane wrapped around a three-cycle of a Calabi-Yau space (for $d=4$) or a $D2$ brane wrapped around a two-cycle of $K3$ (for $d=6$). In the $d=10$ case the D instanton is associated with a pair of integers which are identified, after T duality, with the charge of a D particle threshold bound state and with the winding number of its world line.

The power of the instanton action $(S_N)^{a_d}$ that enters the measure in Eq. (1.5) takes the values $a_d=-7/2$, $-3/2$ and $-1/2$ for $d=10$, 6 and 4, respectively. This factor should emerge from the Jacobian for the change of variables from zero modes to collective coordinates. Although we have not done that calculation explicitly, these values of a_d are consistent with attributing a factor of $S_N^{1/4}$ to each bosonic collective coordinate and $S_N^{-3/8}$ for each fermionic collective coordinate.

Having the exact partition function for all N might be of significance for the zero-dimensional matrix model [3] and other applications of large-N supersymmetric Yang-Mills theory. In that context it is perhaps notable that the $d=10$ expression (1.6) does *not* have a well-defined large-N limit. The special interactions considered in this paper (such as the R^4 term) are interpreted in the Yang-Mills matrix model as a special class of local gauge-invariant correlation functions. These can be expressed as correlations of small Wilson loops that are associated with punctures, or vertex operators, in the string picture.

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APPENDIX

For all values of d considered in this paper the instanton-induced terms in the effective action may be simply summarized by expressing them in terms of a one-loop Feynman diagram with p external states in the T-dual theory on $R^{d-1}\times S^1$ (this is the spirit in which the $d=4$ case was discussed in [45]). Integration over the fermionic modes gives rise to the interaction $\mathcal{P}(\Psi)$ leaving a one-loop amplitude for a scalar field theory that determines the coefficients $\mathcal{F}_{\mathcal{P}}^{(d)}$ which are functions of the moduli. This amplitude is given by

$$A_{\mathcal{P}}=R\pi^{-d/2}\sum_n \int d^{d-1}\mathbf{p}\int \frac{dt}{t}\, t^p e^{-t[p^2+(n-x)^2/R^2+\mu^2]}, \tag{A1}$$

where R is the radius of the (Euclidean) circle, n is the Kaluza-Klein charge and the factor t^p originates from integrating over the proper times of the p vertex operators around the loop. The shift in the integer momentum is because of a nonzero R⊗R Wilson line in the compact direction.

Integration over the loop momentum $\mathbf{p}$ and a Poisson resummation with respect to n gives

$$A_{\mathcal{P}}=R^2\sum_m \int \frac{dt}{t}\, t^{k-d/2}e^{-(\pi^2R^2m^2)/(t)-t\mu^2}e^{2\pi imx}, \tag{A2}$$

where m is the winding number of the world line around the compact dimension. In this form the ultraviolet divergence of the loop amplitude arises in a single zero-winding number $(m=0)$ term. The terms with nonzero m give the instanton corrections that we are interested in here.

The cases under consideration in this appendix are those with $k-d/2=-1$. These one-loop amplitudes contain vertex operator insertions which are ''maximal'' in the sense that each vertex operator absorbs four fermionic zero modes and corresponds to a two-derivative term on a bosonic field—the four vertices of linearized R^4 in $d=10$, the two vertices of $\partial^2\phi\partial^2\phi$ in $d=6$, and the one $\partial^2\phi$ vertex in $d=4$. Note that in the case $d=4$ such a term is related to the corrections of the metric discussed in Sec. IV by an integration by parts as will be seen explicitly at the end of this appendix.

After a change of variables $t\to 1/t$ these amplitudes are given by

$$\begin{aligned}A_{\mathcal{P}}&=R^2\sum_m \int dt\, \exp\left(-t\pi^2R^2m^2-\frac{\mu^2}{t}\right)e^{2\pi imx}\\ &=\pi^{-2}\sum_m \frac{1}{m^2}\int dt\, \exp\left(-t-\frac{(2\pi mR\mu)^2}{4t}\right)e^{2\pi imx}\\ &=\pi^{-2}\sum_m \frac{1}{m^2}\,(2\pi mR\mu)K_{-1}(2\pi|m|R\mu)e^{2\pi imx}.\end{aligned} \tag{A3}$$

In general, the mass μ of the D particle is a function of the moduli of the form $\mu=f(z^i)e^{-\phi_d}$ where ϕ_d is the d-dimensional dilaton. This transforms to ϕ_d' under T duality where

$$2\pi mR\mu=2\pi mRf(z)e^{-\phi_d}=2\pi mf(z)e^{-\phi_d'}=S_m. \tag{A4}$$

Substituting this into Eq. (A3) and expanding the Bessel function for large argument produces a leading instanton contribution of

$$\sum_m \frac{1}{m^2}\,(S_m)^{1/2}e^{-S_m}e^{2\pi imx} \tag{A5}$$

in all cases. This assumes that the D particle circulating in the loop is nondegenerate, which is true for $d=4$ and $d=6$. In the $d=10$ case there are charge-k threshold bound states with masses $\mu_k = k\mu$ that circulate in the loop. The amplitude must therefore be summed over k as well as the winding number m. Writing $N = mk$ the result is

$$A = \pi^{-2} \sum_N \sum_{N|n} \frac{1}{n^2} (S_N) K_{-1}(S_N) e^{2\pi i N x}. \tag{A6}$$

Hence the different behavior of the instanton measure $\mathcal{Z}_N$ for $d = 10$ compared to $d = 4,6$.

The loop amplitude (A2) in the $d = 4$ case corresponds to the one point,

$$\partial^2 \Phi \sum_{N \neq 0} \frac{1}{N^2} |N e^{-\Phi}| K_{-1}(2\pi |N| e^{-\Phi}) e^{2\pi i N x}, \tag{A7}$$

whereas in Sec. IV we reviewed the correction to the metric on the moduli space given in [38] which had the form

$$V e^{-2\Phi} \partial_\mu \Phi \partial^\mu \Phi, \tag{A8}$$

where V is defined in Eq. (4.3). In fact the nonperturbative contribution to Eq. (A8) is equal to Eq. (A7), up to a total derivative which vanishes when integrated. This follows simply from the relation among Bessel functions:

$$\partial_x (x K_{-1}(x)) = -x K_0(x). \tag{A9}$$

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