Renormalization of gravitational self-interaction for wiggly strings

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(Received 8 October 1998; published 8 September 1999)

It is shown that for any elastic string model with energy density U and tension T the divergent contribution from gravitational self-interaction can be allowed for by an action renormalization proportional to $(U-T)^2$. This formula is applied to the important special case of a bare model of the transonic type (characterized by a constant value of the product UT) that represents the macroscopically averaged effect of short-wavelength wiggles on an underlying microscopic model of the Nambu-Goto type (characterized by $U=T$).
[S0556-2821(99)06216-5]

PACS number(s): 98.80.Cq, 04.40.−b, 11.27.+d

I. INTRODUCTION

Although not so important for lightweight cosmic strings, such as may may have been formed later on (for example at the time of electroweak symmetry breaking), gravitational self-interaction is generally supposed to have played an essential role in the evolution of the cosmic strings that may have been formed earlier, at the epoch of grand unified theory (GUT) symmetry breaking. In the kind of scenario [1] originally proposed by Kibble, an initial period during which the main damping mechanism was the friction exerted by the ambient thermal gas would be followed by a period during which the main damping mechanism would have been gravitational, at least for the local kind of strings to be considered here (in contrast with global strings for which axion radiation damping would have been more important).

Nearly all the work that has been done [1] on gravitational self-interaction in cosmic strings has been based on the use of a string model of the simplest kind, namely that of Nambu-Goto, in which (with the speed of light set to 1) the energy density U and the tension T are both equal to a constant m^2, where m is a mass scale that will typically be of the order of the mass associated with the Higgs field responsible for the relevant vacuum symmetry breaking. The use of such a simple model will be justifiable while gravitational self-interaction is important, even in typical cases where the strings are of the current carrying kind whose likely relevance was first pointed out by Witten [2], since, at least at the outset, the currents would be expected to be very weak so that their effects would be relatively negligible.

At a later stage the currents in small loops would intensify as the loops contracted due to radiative energy loss, so that it might become necessary to use a description based on an elastic string model of an appropriate kind [3], in terms of which it is possible to describe oscillations [4,5] about stationary ‘‘vorton’’ states [6] (whose existence would not even be possible if the Nambu-Goto description remained valid). However, by this later stage (except in the case of an electromagnetically neutral current), the main self-interaction mechanism would no longer be gravitational, but electromagnetic. Thus, as long as one has to do with a regime in which gravitation is still the main self-interaction mechanism, a Nambu-Goto description will usually be adequate.

Although adequate in principle, an absolutely precise Nambu-Goto description will seldom be feasible in numerical simulations, due to the enormous amount of information that would be required for following the detailed evolution of small scale wiggles. Some kind of approximation will therefore be needed in pratice. Following the remark by Shellard and Allen [7] that in the presence of small scale wiggles the macroscopically averaged value of the energy density U would exceed its Nambu-Goto value m^2, I observed [8] that the tension T would be correspondingly diminished and proposed as an approximation the use of a model in which the product of energy density and tension remained constant:

$$UT=m^4. \tag{1}$$

This relation was confirmed for a special subclass of wiggle modes by Vilenkin [9] and more recently for unrestricted wiggle modes by Martin [10]. A particularly attractive feature of this model is the property of being transonic, in the sense that the propagation speed

$$c_E=\sqrt{T/U} \tag{2}$$

of its extrinsic (tranverse) perturbations is the same as the speed

$$c_L=\sqrt{-dT/dU} \tag{3}$$

of its longitudinal (sound-type) modes. This makes it possible to prove [11] that such a model can match the long-term evolution of the string with any desired acuracy (depending on the resolution) and without any accumulative error buildup, at least in flat spacetime where the dynamical equations are exactly integrable.

For a practical description of cosmic strings in circumstances where the main kind of self-interaction is gravitational, it is therefore this transonic model that will commonly be most appropriate. The purpose of this article is to consider the way this model will need to be modified to allow for the dominant effect of the self-interaction, which will be divergent.

As in the simpler case of electromagnetic self-interaction, gravitational self-interaction in a four-dimensional spacetime background will give rise in point particle models to pole-type singularities and in string models to logarithmic singularities, which need to be regularized by the use of an ‘‘ultraviolet'' cutoff. In order to do this in a systematic manner, I have developed a simple technical formalism [12] whose application to the electromagnetic self-interaction gives results that agree with the conclusions of earlier approaches, while providing a more convenient means of dealing with applications to particular problems such as oscillations of a conducting string loop about a stationary vorton state [13]. The more recent application of this formalism to the linearized gravitational case [14] has, however, given results [15] that deviate from what had previously been obtained [1] in the only case that had been considered previously, namely that of a Nambu-Goto string, for which the divergent part simply vanishes. A detailed examination of this particular case [16] has shown that previous assertions to the contrary [1] were effectively based on the neglect of terms that would indeed be relatively small when due to high-frequency gravitational radiation from a distant source, but that in the case of local self-gravitation are of the same order as the other terms, which they finally cancel.

What will be done here is to apply the general formula [15] for the divergent part of the self-gravitational interaction of a string to the nontrivial case of a generic elastic string model and, in particular, to the case of the transonic string model, for which it does not vanish. The nonvanishing result for the transonic string model is entirely consistent with its interpretation as a smoothed approximation to a wiggly Nambu-Goto model for which the corresponding short-range self-gravitational contribution vanishes. The nonvanishing short-range contribution in the approximate description represents the finite intermediate-range contribution in the underlying Nambu-Goto model. Thus the treatment provided here takes care of everything except the very-long-range part of the self-interaction, which will be dynamically negligible in the short run, though it is of course very important in the long run since it is the part that is ultimately responsible for the radiative damping.

Unlike the dissipative long-range part, the dominant contribution considered here is strictly conservative: one of the main results of this work is the demonstration that for a generic (not just a transonic) elastic string model the divergent self-gravitational stress energy contribution given (''on shell'') by the new formula [15] is derivable from a corresponding (''off-shell'') action contribution that is precisely the same as what is obtained as the four-dimensional specialization of a more general (higher-dimensional) action formula that has recently been derived using an entirely different approach by Buonanno and Damour [17]. The net effect is thus describable as an action renormalization, whose effect is trivial in the case of a Nambu-Goto model for which (consistently with what was suggested by exact analytic considerations in static configurations [18]) it simply vanishes, but nontrivial in the more general string models considered here.

II. GRAVITATIONAL FORCE DENSITY

Since the cosmic strings to which this work applies will be characterized by a gravitational coupling constant that is very small, $Gm^2 \lesssim 10^{-6}$ (where G is Newton's constant) even for the heavyweight case of GUT strings, it is sufficient to base the analysis on linearized gravitational theory, as expressed in terms of a small but perhaps rapidly varying perturbation $h_{\mu\nu} = \delta g_{\mu\nu}$ of a slowly varying cosmological background metric $g_{\mu\nu}$. Moreover, for the purpose of analyzing the short-range self-interaction, it will sufficient to neglect the background curvature altogether, i.e., to take the unperturbed four-dimensional spacetime metric $g_{\mu\nu}$ to be flat, so that subject to the usual [18] DeDonder gauge condition $\nabla^\mu h_{\mu\nu} = \frac{1}{2}\nabla_\nu h^\mu_\mu$, the linearized Einstein equations for the Eulerian metric perturbation will reduce to the well-known form

$$\nabla_\sigma \nabla^\sigma h^{\mu\nu} = -8\pi G(2\hat{T}^{\mu\nu} - \hat{T}_\sigma{}^\sigma g^{\mu\nu}), \tag{4}$$

where $\hat{T}^{\mu\nu}$ is the stress momentum energy density tensor of the source.

The problem of ultraviolet divergences for point particle or string models arises because in these cases the relevant source densities are not regular functions, but Dirac-type distributions that vanish outside the relevant one- or two-dimensional world sheets. In the case of a string with local world sheet embedding given by $x^\mu = \overline{x}^\mu\{\sigma\}$ in terms of intrinsic coordinates σ^i $(i=0,1)$, so that the induced surface metric will have the form $\gamma_{ij} = g_{\mu\nu}\overline{x}^\mu{}_{,i}\overline{x}^\nu{}_{,j}$, the relevant source distribution will be expressible using the terminology of Dirac delta ''functions'' in the form

$$\hat{T}^{\mu\nu} = \|g\|^{-1/2} \int \overline{T}^{\mu\nu} \delta^4[x - \overline{x}\{\sigma\}] \|\gamma\|^{1/2} d^2\sigma, \tag{5}$$

where $|\gamma|$ is the determinant of the induced metric and the surface stress-energy density $\overline{T}^{\mu\nu}$ is a *regular* tensorial function on the world sheet (but undefined off it).

In the simple elastic models considered here, the only internal field on the string will be a surface current density $\overline{c}^\mu = \varepsilon^{ij}\overline{x}^\mu{}_{,i}\psi_{,j}$, where ψ is a scalar stream function on the world sheet (which will be a free variable in the variation formulation described below), using the notation $\varepsilon^{ij} = -\varepsilon^{ji}$ for the antisymmetric world sheet measure tensor that is specified (modulo a choice of sign representing an orientation convention) as the square root of the induced metric, i.e., $\varepsilon^{ik}\varepsilon_{kj} = \gamma^i{}_j$. If there were a nonzero charge coupling constant q, as supposed in Witten's theory [2] of superconducting strings, then this would correspond to an electric surface current density $\overline{j}^\mu = q\overline{c}^\mu$. However, the present discussion is concerned just with the early regime in which effects of electromagnetic coupling are negligible compared with those of gravitation. This means that in the relevant action for the ''bare'' string model (i.e., the non-self-interacting limit), as given by an integral of the form $\mathcal{I} = \int \overline{\mathcal{L}} \|\gamma\|^{1/2} d^2\sigma$, the specification of the relevant Lagrangian scalar $\overline{\mathcal{L}}$ on the world sheet will be given by a master function, Λ that depends only on the undifferentiated background metric and the gradient of the stream function ψ, according the formula

$$\overline{\mathcal{L}} = \Lambda + \tfrac{1}{2}\overline{T}^{\mu\nu} h_{\mu\nu}, \tag{6}$$

in which the coefficient for the linearized gravitational adjustment term here is the surface stress-energy tensor that specifies the gravitational source in Eq. (4), which is given by $\overline{T}^{\mu\nu}=2\|\gamma\|^{-1/2}\partial(\Lambda\|\gamma\|^{1/2})/\partial g_{\mu\nu}$.

Since the ensuing field equations will evidently involve gradients of the stress-energy tensor, their formulation will require the introduction of the appropriately defined hyper-Cauchy tensor (a relativistic generalization of the Cauchy elasticity tensor of classical mechanics), which is defined by $\overline{\mathcal{C}}^{\mu\nu\rho\sigma}=\|\gamma\|^{-1/2}\partial(\overline{T}^{\rho\sigma}\|\gamma\|^{1/2})/\partial g_{\mu\nu}$. In terms of this quantity, the dynamical equations obtained from the Lagrangian (6) can be shown [14] to be expressible in the standard form

$$\overline{\nabla}_\nu \overline{T}^{\mu\nu}=f_g{}^\mu, \tag{7}$$

where the effective gravitational force density vector is given by

$$f_g{}^\mu=\tfrac{1}{2}\overline{T}^{\nu\sigma}\nabla^\mu h_{\nu\sigma}-\overline{\nabla}_\nu(\overline{T}^{\nu\sigma}h_\sigma{}^\mu+\overline{\mathcal{C}}^{\mu\nu\rho\sigma}h_{\rho\sigma}), \tag{8}$$

using the notation $\overline{\nabla}_\mu=\eta_\mu{}^\nu\nabla_\nu$ for the tangentially projected gradient operator, where $\eta_\mu{}^\nu$ is the tangential projection tensor, i.e., the index-lowered form of the first fundamental tensor of the world sheet, which is obtained simply by mapping its internal metric onto the spacetime background according to the formula $\eta^{\mu\nu}=\gamma^{ij}\overline{x}^\mu{}_{,i}\overline{x}^\nu{}_{,j}$.

III. RENORMALIZATION

The problem with the application of Eq. (8) is of course that the linearized gravitational field will consist not just of a well-behaved long-range contribution, $\tilde{h}_{\mu\nu}$ say, but also of a divergent short-range self-interaction contribution $\hat{h}_{\mu\nu}=h_{\mu\nu}-\tilde{h}_{\mu\nu}$, which needs to be appropriately regularized in the manner recently described in the analogous electromagnetic case [12]. This routine procedure leads to a result that is proportional to the relevant source in Eq. (4), which gives

$$\hat{h}_{\mu\nu}=2G\hat{l}(2\overline{T}_{\mu\nu}-\overline{T}_\sigma{}^\sigma g_{\mu\nu}), \tag{9}$$

where, as usual for a string self-interaction in four dimensions, the proportionality factor has the form

$$\hat{l}=\ln\{\Delta^2/\delta_*^2\} \tag{10}$$

in terms of an ''ultraviolet'' cutoff length scale δ_* representing the effective thickness of the string and a much larger ''infrared'' cutoff Δ given by a length scale characterizing the large-scale geometry of the string configuration. As pointed out in the electromagnetic case [12], the corresponding regularized value of the gradient of such a divergent self-field will be obtainable from the regularized self-field by application of the regularized gradient operator defined by

$$\hat{\nabla}_\mu=\overline{\nabla}_\mu+\tfrac{1}{2}K_\mu, \tag{11}$$

where K_μ is the world sheet curvature vector that is obtainable as the surface divergence of the fundamental (tangential projection) tensor, $K_\mu=\overline{\nabla}_\nu\eta^\nu_\mu$.

When the corresponding divergent self-force contribution is evaluated substituting Eq. (9) and (11) into Eq. (8), the result turns out, rather remarkably, to be describable as a renormalization of the stress-energy tensor, since one obtains [15] a regularized self-force that is expressible as world sheet divergence

$$\hat{f}^\mu_g=-\overline{\nabla}_\nu\hat{T}_g{}^{\mu\nu}, \tag{12}$$

in which the relevant stress momentum energy density contribution from the gravitational self-interaction has the form

$$\hat{T}_g{}^{\mu\nu}=\hat{h}_\sigma{}^\mu\overline{T}^{\nu\sigma}-\tfrac{1}{4}\hat{h}_{\rho\sigma}\overline{T}^{\rho\sigma}\eta^{\mu\nu}+\hat{h}_{\rho\sigma}\overline{\mathcal{C}}^{\rho\sigma\mu\nu}. \tag{13}$$

It is also to be observed that this ''on-shell'' self-gravitational stress energy contribution is obtainable from a corresponding ''off-shell'' self-gravitational action contribution given by

$$\hat{\Lambda}_g=\tfrac{1}{4}\overline{T}^{\mu\nu}\hat{h}_{\mu\nu}=\tfrac{1}{2}G\hat{l}(2\overline{T}_{\mu\nu}\overline{T}^{\mu\nu}-\overline{T}_\mu{}^\mu\overline{T}_\nu{}^\nu). \tag{14}$$

This provides a regularized treatment in which the original world sheet Lagrangian $\mathcal{L}$ is replaced by a regularized Lagrangian $\tilde{\mathcal{L}}=\tilde{\Lambda}+\tfrac{1}{2}\overline{T}^{\mu\nu}\tilde{h}_{\mu\nu}$ involving only the well-behaved part $\tilde{h}_{\mu\nu}$ of the gravitational field, whereby the divergent part $\hat{h}_{\mu\nu}$ is absorbed into a renormalized master function given by

$$\tilde{\Lambda}=\Lambda+\hat{\Lambda}_g. \tag{15}$$

The consistency of this treatment has been neatly confirmed by an independent investigation in which, by working entirely at the level of the ''off-shell'' action in a space-time of arbitrary dimension, Buonanno and Damour [17] have recently obtained a general self-interaction formula whose specialization to the case of gravitation in four-dimensions is in precise agreement with the result (14) obtained here.

For any simple elastic string model of the kind considered here, the master function Λ will depend just on the scalar magnitude that is specifiable [3] as $\chi=\overline{c}^\mu\overline{c}_\mu=-\gamma^{ij}\psi_{,i}\psi_{,j}=-p_\mu p^\mu$ where the relevant momentum vector is defined by $p^\mu=\overline{\nabla}^\mu\psi=\overline{x}^\mu{}_{,i}\gamma^{ij}\psi_{,j}$. Using the notation $\Lambda'=d\Lambda/d\chi$, it can be seen that the regularized self-field will be given by

$$\hat{h}^{\mu\nu}=4G\hat{l}(2\Lambda' p^\mu p^\nu+\chi\Lambda' g^{\mu\nu}-\Lambda\perp^{\mu\nu}), \tag{16}$$

using the notation $\perp_\nu{}^\mu=g_\nu{}^\mu-\eta_\nu{}^\mu$ for the (rank-2) world sheet orthogonal projection tensor. The corresponding self-gravitational action contribution (14) will be expressible as

$$\hat{\Lambda}_g=\tfrac{1}{2}G\hat{l}(U-T)^2=2G\hat{l}(\chi\Lambda')^2. \tag{17}$$

IV. EFFECT ON STRESS TENSOR AND PROPAGATION SPEEDS

In analogy with the standard formula

$$\overline{T}^{\mu\nu}=\Lambda\,\eta^{\mu\nu}+2\Lambda' p^\mu p^\nu, \tag{18}$$

for the ''bare'' surface stress-energy tensor, the corresponding self-gravitational stress energy tensor (13) will be obtainable in the form

$$\hat{T}_g{}^{\mu\nu}=\hat{\Lambda}_g\,\eta^{\mu\nu}+2\hat{\Lambda}'_g p^\mu p^\nu. \tag{19}$$

If the current is timelike—as can be assumed without loss of generality in the wiggly Nambu-Goto string approximation—so that we have $\chi\leqslant 0$, then the energy density and tension will be given for the bare model by $U=-\Lambda$ and $T=2\chi\Lambda'-\Lambda$, while the corresponding extrinsic (wiggle-type) and longitudinal (sound-type) perturbations speeds c_E and c_L will be given [3] by

$$c_E^2=1-2\chi\Lambda'/\Lambda, \quad c_L^2=1+2\chi\Lambda''/\Lambda'. \tag{20}$$

An elastic string state is describable as supersonic, transonic, or subsonic, according to whether the difference

$$c_E^2-c_L^2=-2\chi(\ln\{\Lambda\Lambda'\})' \tag{21}$$

is positive, zero, or negative. In terms of these quantities the corresponding renormalized energy density (for a given value of the current magnitude as specified by χ) will be obtainable directly from the renormalized action (15) as

$$\tilde{U}=-\tilde{\Lambda}=U-\tfrac{1}{2}G\hat{l}UT(c_E^{-1}-c_E)^2. \tag{22}$$

However, the evaluation of the corresponding renormalized tension is not so quite so simple: it works out to be

$$\tilde{T}=T+\tfrac{1}{2}G\hat{l}UT(c_E^{-1}-c_E)^2(1+2c_L^2). \tag{23}$$

V. CASE OF THE TRANSONIC WIGGLY STRING MODEL

In the exact Nambu-Goto case [16], the master function is just a constant, $\Lambda=-m^2$, where (in units with the Dirac Planck constant $\hbar$ set to unity) the parameter m is the relevant Kibble mass scale, which is a constant that can be expected to be of the same order of magnitude as the Higgs mass scale associated with the underlying symmetry breaking for a string of the ordinary ''cosmic'' kind, representing a vortex-type defect of the vacuum. In this case one simply obtains $\Lambda'=0$, so the divergent short-range self-gravitational contribution vanishes.

However, for the purpose of a course-grained description on a larger scale, it is appropriate to use the transonic model [8,9] to represent the smoothened average over short-wavelength wiggles in the underlying microscopic Nambu-Goto model. For this transonic string model, the relevant master function has the form

$$\Lambda=-m\sqrt{m^2-\chi}, \tag{24}$$

with nonzero derivative

$$\Lambda'=-m^2/2\Lambda. \tag{25}$$

This gives a nonzero self-gravitional contribution attributable to an interaction at intermediate range, i.e., distances large compared with the wiggle wavelength in the underlying macroscopic model, but small compared with the smoothening length on which the macroscopic description is based. A noteworthy example of the application of this model is to the case of wiggles that are of purely thermal origin, for which the state parameter χ will be given [8,19] as a function of the relevant temperature Θ by the formula

$$\chi=\frac{-2\pi m^2\Theta^2}{3m^2-2\pi\Theta^2}, \tag{26}$$

from which it can be seen that corresponding wiggle propagation speed will be given by

$$c_E^2=1-\frac{2\pi\Theta^2}{3m^2}. \tag{27}$$

It is evident that the special transonicity property

$$c_L=c_E \tag{28}$$

of the ''bare'' model will not survive in the renormalized model, as obtained using Eq. (24) from Eqs. (15) and (17), for which (still working just to linear order in the gravitational coupling) the difference between the squared propagation speeds is found to be given by

$$\widetilde{c_E^2}-\widetilde{c_L^2}=Gm^2\hat{l}(c_E^{-1}-c_E)[(1+c_E^2)^2+\tfrac{1}{2}(1-c_E^2)^2]. \tag{29}$$

The manifest positivity (since $0<c_E<1$) of this result shows that the ''dressed'' gravitationally self-interacting wiggly string model will be of supersonic type.

VI. ORDERS OF MAGNITUDE

The regime of applicability of the foregoing analysis is of course subject to limitations. To start with, for the validity of the linearized gravitation equation (4) on which the entire analysis depends, the weak-coupling condition

$$Gm^2\hat{l}\ll 1 \tag{30}$$

must be satisfied. This requirement is not very restrictive, because even the heaviest kind cosmic strings that are commonly considered in cosmological applications, namely those arising from GUT symmetry breaking, are characterized by $Gm^2\approx 10^{-6}$. For other kinds, such as those arising from electroweak symmetry breaking, the value of Gm^2 will be smaller still. The smallness of Gm^2 will be partially counterbalanced by the fact that the regularization factor $\hat{l}$ will be large compared with unity, but since, according to Eq. (10), it arises as a logarithm, it can never be extremely large. In nearly all cases that are likely to arise in practice, it will satisfy $\hat{l} \ll 10^2$, so the requirement (30) will be satisfied by a large margin.

Since the rigorous justification [11] of the description of the macroscopic effect of the wiggles by the simple elastic model (24) depends on the supposition that self-intersections of the string can be neglected (since if loop formation were important a more elaborate nonelastic model would be needed), the validity of the model as a precise representation is limited to the regime in which the effective energy density of the wiggles, as formally defined by

$$\varepsilon = \frac{U-T}{2} \simeq U - m^2, \tag{31}$$

is small compared with the intrinsic energy density of the string, i.e.,

$$\varepsilon \ll m^2, \tag{32}$$

which is interpretable in the thermal case as meaning

$$\Theta \ll m. \tag{33}$$

Since this is equivalent to the restriction

$$1 - c_E^2 \ll 1, \tag{34}$$

it can be seen that the difference (29) will be given approximately by the simple formula

$$\widetilde{c_E^2} - \widetilde{c_L^2} \simeq 8G\hat{l}\varepsilon. \tag{35}$$

In an application of this kind, the magnitude, λ, say, typifying the wavelength of the wiggles over which the averaging is taken will provide an appropriate choice for the infrared cutoff; i.e., it will be natural to take $\Delta \approx \lambda$. Similarly, the magnitude, α, say, characterizing the amplitude of the wiggles will provide the corresponding value for the ultraviolet cutoff, which will thus be given by $\delta_* \approx \alpha$, provided that, as will usually be the case, the amplitude of the wiggles is not even smaller than the microscopic string radius, r, say, in which case the latter would itself provide the relevant ultraviolet cutoff. When the string is of the usual ''cosmic'' variety, representing an underlying vortex-type defect of the vacuum, one expects the radius to be of the same order of magnitude as the Compton wavelength associated with the relevant Kibble mass; i.e., one expects to have $r \approx m^{-1}$. In most applications of interest, the relevant amplitude α will be considerably larger than this, and even for wiggles of purely thermal origin it will never be smaller: for the wiggles produced by a given temperature Θ, as given by Eq. (26), one can estimate the relevant magnitudes as $\lambda \approx \Theta^{-1}$ and $\alpha \approx m^{-1}$; i.e., independently of the temperature the relevant amplitude will be of the same order as the cosmic string radius, $\alpha \approx r$. This means that, in the usual physical applications, it will generally be possible to take the relevant cutoff ratio to be given by $\Delta/\delta_\star \approx \lambda/\alpha$.

Since the effective energy density ε contributed by small wiggles of wavelength λ and amplitude α can be roughly estimated as $\varepsilon \approx m^2\alpha^2/\lambda^2$, we see that the squared cutoff ratio appearing in the logarithm in Eq. (10) will be given by the corresponding rough estimate $\Delta^2/\delta_*^2 \approx m^2/\varepsilon$ and, hence, that a reasonably accurate description should be obtainable by taking the regularization factor itself to be given by an approximation of the form

$$\hat{l} \simeq \ln\left\{\frac{m^2}{\langle\varepsilon\rangle}\right\}, \tag{36}$$

where $\langle\varepsilon\rangle$ is a constant chosen as some suitably weighted mean value of the energy density $\varepsilon = m^2\chi/2\Lambda$ in the string segment under consideration. The requirement (32) that the wiggle energy density should be small compared with the intrinsic energy of the string implies that the ensuing factor $\hat{l}$ will be reasonably large compared with unity, and the consideration that the dependence is logarithmic means that the result will be insensitive to the details of the particular prescription chosen to specify $\langle\varepsilon\rangle$. In order to obtain higher accuracy, one might be tempted to replace the fixed mean value $\langle\varepsilon\rangle$ in Eq. (36) by the variable local value of ε, which, by Eq. (17), would be equivalent to taking the self-gravitational action adjustment to be

$$\hat{\Lambda}_g \simeq 2G\varepsilon^2 \ln\left\{\frac{m^2}{\varepsilon}\right\}, \tag{37}$$

with

$$\varepsilon = \frac{-m\chi}{2\sqrt{m^2-\chi}}. \tag{38}$$

However, the appearance of improvement provided by such use of a variable rather than a constant value for the renormalization factor $\hat{l}$ is rather illusory, since the preceding demonstration of renormalizability—as embodied in the formulas (12), (13), and (14)—was dependent on the postulate that $\hat{l}$ should be constant. Moreover, it can easily be checked explicitly that the only effect of the use of the apparently more precise formula (37) on the final formula (35) will be to replace the factor $\hat{l}$ by $\hat{l} - \frac{3}{2}$. This adjustment would be significant only if ε were so large as to be comparable with m^2, so that the accuracy of the treatment would in any case be affected by other complicating processess such as loop formation due to self-intersections.

ACKNOWLEDGMENTS

The author wishes to thank Xavier Martin, Paul Shellard, and Alex Vilenkin for earlier discussions about the treatment of string wiggles and to thank Richard Battye, Thibault Damour, Alejandro Gangui, Gary Gibbons, and Patrick Peter for more recent discussions about regularization and renormalization.

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