Sensitivity of wideband detectors to quintessential gravitons

D. Babusci*
INFN-Laboratori Nazionali di Frascati, I-00044 Frascati, Italy

M. Giovannini†
Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
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There is no reason why the energy spectra of the relic gravitons amplified by the pumping action of the background geometry should not increase at high frequencies. A typical example of this behavior is quintessential inflation where the slopes of the energy spectra can be either blue or mildly violet. In comparing the predictions of scenarios leading to blue and violet graviton spectra we face the problem of correctly deriving the sensitivities of the interferometric detectors. Indeed the expression of the signal-to-noise ratio not only depends upon the noise power spectra of the detectors but also upon the spectral form of the signal and, therefore, one can reasonably expect that models with different spectral behaviors will produce different signal-to-noise ratios. By assuming monotonic (blue) spectra of relic gravitons we will give general expressions for the signal-to-noise ratio in this class of models. As an example we study the case of quintessential gravitons. The minimum achievable sensitivity to $h_0^2\,\Omega_{\rm GW}$ of different pairs of detectors is computed, and compared with theoretical expectations. [S0556-2821(99)04718-9]

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I. INTRODUCTION

Gravitational wave astronomy, experimental cosmology, and high energy physics will soon experience a boost thanks to the forthcoming interferometric detectors. From a theoretical point of view it is then interesting to compare our theoretical expectations or speculations with the foreseen sensitivities of the various devices in a frequency range which complements and greatly extends the information we can derive from the analysis of the microwave sky and of its temperature fluctuations.

By focusing our attention on relic gravitons of primordial origin we can say that virtually every variation in the time evolution of the curvature scale can imprint important information on the stochastic gravitational wave background [1]. The problem is that the precise evolution of the curvature scale is not known. Different cosmological scenarios, based on different physical models of the early Universe, may lead to different energy spectra of relic gravitons and this crucial theoretical indetermination can affect the expected signal.

Of particular interest seems to be the case where the logarithmic energy density of the relic gravitons (in critical units) grows [2,3] in the frequency region explored by the interferometric detectors (i.e., approximately between few Hz and 10 kHz) [4–8]. In this range we can parametrize the energy density of the relic gravitons $\rho_{\rm GW}$ at the present time η_0 as

$$\Omega_{\rm GW}(f,\eta_0)=\frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f}=\overline{\Omega}(\eta_0)q(f,\eta_0), \qquad (1.1)$$

where $\overline{\Omega}(\eta_0)$ denotes the typical amplitude of the spectrum and $q(f,\eta_0)$ is a monotonic function of the frequency at least in the interval $1\,{\rm Hz}\lesssim f\lesssim 10\,{\rm kHz}$. Both $\overline{\Omega}(\eta_0)$ and $q(f,\eta_0)$ can depend on the parameters of the particular model. The assumption that $q(f,\eta_0)$ is monotonic can certainly be seen as a restriction of our analysis, but, at the same time, we can notice that the models with growing logarithmic energy spectra which were discussed up to now in the literature fit in our choice for $q(f,\eta_0)$. Within the parametrization defined in Eq. (1.1) we will be discussing the cases where the spectral slope α [i.e., $\alpha=d\ln q(f,\eta_0)/d\ln f$] is either blue (i.e., $0<\alpha\lesssim 1$) or violet (i.e., $\alpha>1$). In general we could have also the case $\alpha<0$ (red spectra) and $\alpha=0$ (flat spectrum). Flat spectra have been extensively studied in the context of ordinary inflationary models [9] and in relation to cosmic string models [10].

Blue and violet spectra are physically peculiar since they are typically produced in models which are different from the ones leading to flat spectra. In quintessential inflationary models [11] the logarithmic energy spectra are typically blue [12]. This is due to the fact that in this class of models an ordinary inflationary phase is followed by an expanding phase whose dynamics is driven by an effective equation of state which is stiffer than radiation [13]. Since the equation of state (after the end of inflation) is stiffer than the one of radiation, then the Universe will expand slower than in a radiation dominated phase and, therefore, α turns out to be at most one (up to logarithmic corrections).

In string cosmological models [14] the graviton spectra can be either blue (if the physical scale corresponding to a present frequency of 100 Hz went out of the horizon during the string phase) or violet (if the relevant scale crossed the horizon during the dilaton driven phase).

The purpose of this paper is to analyze the sensitivity of pairs of interferometric detectors to blue and violet spectra of relic quintessential gravitons. The reason for such an exercise is twofold. On one hand violet and blue spectra, owing

*Electronic address: Danilo.Babusci@lnf.infn.it
†Electronic address: giovan@cosmos2.phy.tufts.edu

to their growth in frequency, might provide signals which are larger than in the case of flat inflationary spectra. On the other hand the sensitivity to blue spectra from quintessential inflation can be different from the one computed in the case of flat spectra from ordinary inflationary models. Indeed, it is sometimes common practice to compare the theoretical energy density of the produced gravitons with the sensitivity of various interferometers to a flat spectrum. This is, strictly speaking, arbitrary even if, sometimes this procedure might lead to correct order of magnitude estimates.

In order to illustrate qualitatively this point let us consider the general expression of the signal-to-noise ratio (SNR) in the case of correlation of two detectors of arbitrary geometry for an observation time T. By assuming that the intrinsic noises of the detectors are stationary, Gaussian, uncorrelated, much larger in amplitude than the gravitational strain, and statistically independent on the strain itself, one has [15–18][1]

$$\mathrm{SNR}^2 = \frac{3H_0^2}{2\sqrt{2}\,\pi^2} F\,\sqrt{T}\left\{ \int_0^\infty df\,\frac{\gamma^2(f)\,\Omega_{\mathrm{GW}}(f)}{f^6\,S_n^{(1)}(f)\,S_n^{(2)}(f)}\right\}^{1/2} \tag{1.2}$$

(H_0 is the present value of the Hubble parameter and F depends upon the geometry of the two detectors; in the case of the correlation between two interferometers $F=2/5$). In Eq. (1.2), $S_n^{(k)}(f)$ is the (one-sided) noise power spectrum of the kth ($k=1,2$) detector, while $\gamma(f)$ is the overlap reduction function [17,18] which is determined by the relative locations and orientations of the two detectors. This function cuts off (effectively) the integrand at a frequency $f\sim 1/2d$, where d is the separation between the two detectors.

From Eq. (1.2) we can see that the frequency dependence of the signal directly enters in the determination of the SNR and, therefore, we can expect different values of the integral depending upon the relative frequency dependence of the signal and of the noise power spectra associated with the detectors. Hence, in order to get precise information on the sensitivities of various detectors to blue and violet spectra we have to evaluate the SNR for each specific model at hand.

The analysis of the SNR is certainly compelling if we want to confront quantitatively our theoretical conclusions with the forthcoming data. Owing to the difference among the various logarithmic energy spectra of the relic gravitons we can wonder if different detector pairs can be more or less sensitive to a specific theoretical model. We will try, when possible, to state our conclusions in such a way that our results could be used not only in the specific cases discussed in the present paper.

If the graviton spectra increase in frequency we can expect, on a general ground, that the higher the frequency the larger will be the signal. This feature is of course peculiar also in the case of the spectra of quintessential gravitons. Therefore, we can expect that electromagnetic detectors of gravitational waves might also play an interesting role in the context of the scenarios discussed in the present paper. The feasibility study of electromagnetic detectors in the context of relic gravitons has been discussed for the first time in Ref. [19] more than twenty years ago. As we will stress in our study, perhaps the ideas of Ref. [19] should be explored again in light of the most recent theoretical and technological developments.

The plan of the paper is the following. In Sec. II we will review the basic features of blue spectra arising in quintessential inflationary models. In Sec. III we will set up the basic definitions and conventions concerning the evaluation of the SNR. In Sec. IV we will be mainly concerned with the analysis of the achievable sensitivities to some specific theoretical model. Section V contains our concluding remarks.

II. BLUE AND VIOLET GRAVITON SPECTRA

A. Basic bounds

Blue and violet logarithmic energy spectra of relic gravitons are phenomenologically allowed [20]. At low frequencies the most constraining bound [21] comes from the Cosmic Background Explorer (COBE) observations [22] of the first (thirty) multipole moments of the temperature fluctuations in the microwave sky which implies that $h_0^2\,\Omega_{\mathrm{GW}}(f_0,\eta_0)$ has to be smaller than 6.9×10^{-11} for frequencies of the order of H_0. At intermediate frequencies (i.e., $f_{\mathrm{p}}\sim10^{-8}$ Hz) the pulsar timing measurements [23] imply that $\Omega_{\mathrm{GW}}(f_{\mathrm{p}},\eta_0)$ should not exceed 10^{-8}. In order to be compatible with the homogeneous and isotropic nucleosynthesis scenario [24,25] we should require that

$$h_0^2\int\ \Omega_{\mathrm{GW}}(f,\eta_0)\ d\ln f < 0.2\,\Omega_\gamma(\eta_0)\,h_0^2 \simeq 5\times10^{-6}, \tag{2.1}$$

where $\Omega_\gamma(\eta_0)=2.6\times10^{-5}h_0^{-2}$ is the fraction of critical energy density in the form of radiation at the present observation time. In Eq. (2.1) the integral extends over all the modes present inside the horizon at the nucleosynthesis time. In the case of blue and violet logarithmic energy spectra the COBE and pulsar bounds are less relevant than the nucleosynthesis one and it is certainly allowed to have growing spectra without conflicting with any of the bounds.[2]

[1]Notice that, with this definition, the SNR turns out to be the square root of the one used in Refs. [15–18]. The reason for our definition lies in the remark that the cross correlation between the outputs $s_{1,2}(t)$ of the detectors is defined as

$$S=\int_{-T/2}^{T/2}dt\int_{-T/2}^{T/2}dt'\,s_1(t)\,s_2(t')\,Q(t,t'),$$

where Q is a filter function. Since S is quadratic in the signals, with the usual definitions, it contributes to the SNR squared.

[2]Notice that the nucleosynthesis bound refers to the case where the underlying nucleosynthesis model is homogeneous and isotropic. The presence of magnetic fields and/or matter-antimatter fluctuations can slightly alter the picture [26,27].

B. Quintessential spectra

Recent measurements of the red-shift luminosity relation in type Ia supernovae [28] suggest the presence of an effective cosmological term whose energy density can be as large as 0.8 in critical units. Needless to say that this energy density is huge if compared with cosmological constant one would guess, for instance, from electroweak (spontaneous) symmetry breaking, i.e., $\rho_{\Lambda}\sim(250\text{GeV})^4$. In order to cope with this problem various models have been proposed [29] and some of them rely on the existence of some scalar field (the quintessence field) whose effective potential has no minimum [30]. Therefore, according to this proposal the evolution of the quintessence field is dominated today by the potential providing then the wanted (time-dependent) cosmological term. In the past the evolution of the quintessence field is in general not dominated by the potential. The crucial idea behind quintessential inflationary models is the identification of the inflaton ϕ with the quintessence field [11]. Therefore, the inflaton-quintessence potential $V(\phi)$ will lead to a slow-rolling phase of de Sitter type for $\phi<0$ and it will have no minimum for $\phi>0$. Hence, *after* the inflationary epoch (but *prior to* nucleosynthesis) the Universe will be dominated by $\dot{\phi}^2$. This means, physically, that the effective speed of sound of the sources driving the background geometry during the post-inflationary phase will be drastically different from the one of radiation (i.e., $c_s=1/\sqrt{3}$ in natural units) and it will have a typical stiff form (i.e., $c_s=1$). The fact that in the post-inflationary phase the effective speed of sound equals the speed of light has important implications for the gravitational wave spectra as it was investigated in the past for a broad range of equations of state stiffer than radiation (i.e., $1/\sqrt{3}<c_s<1$) [12]. The conclusion is that if an inflationary phase is followed by a phase whose effective equation of state is stiffer than radiation, then, the high frequency branch of the graviton spectra will grow in frequency. The tilt depends upon the speed of sound and it is, in our notations,

$$\alpha=\frac{6c_s^2-2}{3c_s^2+1}. \tag{2.2}$$

We can immediately see that for all the range of stiff equations of state (i.e., $1/\sqrt{3}<c_s<1$) we will have that $0<\alpha<1$. The case $\alpha=0$ corresponds to $c_s=1/\sqrt{3}$. This simply means that if the inflationary phase is immediately followed by the ordinary radiation-dominated phase the spectrum will be (as we know very well) flat. The case $c_s=1$ is the most interesting for the case of quintessential inflation. In this case the tilt is maximal (i.e., $\alpha=1$). Moreover, a more precise calculation [12,13] shows that the graviton spectrum is indeed logarithmically corrected as

$$q(f,\eta_0)=\frac{f}{f_1}\ln^2\left(\frac{f}{f_1}\right). \tag{2.3}$$

It is amusing to notice that this logarithmic correction occurs only in the case $c_s=1$ but not in the case of the other stiff (post-inflationary) background. The typical frequency $f_1(\eta_0)$ appearing in Eq. (2.3) is given, today, by

$$f_1(\eta_0)=1132N_s^{-1/4}\left(\frac{g_{\rm dec}}{g_{\rm th}}\right)^{1/3}\text{GHz}. \tag{2.4}$$

Apart from the dependence upon the number of relativistic degrees of freedom (i.e., $g_{\rm dec}=3.36$ and $g_{\rm th}=106.75$) which is a trivial consequence of the redsfhit, $f_1(\eta_0)$ also depends upon N_s which is the number of (minimally coupled) scalar degrees of freedom present during the inflationary phase. The amplitude of the spectrum depends upon N_s as

$$\overline{\Omega}(\eta_0)=\frac{1.64\times10^{-5}}{N_s}. \tag{2.5}$$

The reason for the presence of N_s is that all the minimally coupled scalar degrees of freedom present during the inflationary phase will be amplified sharing approximately the same spectrum of the two polarizations of the gravitons [31]. The main physical difference is that the N_s scalars are directly coupled to fermions and, therefore, they will decay and thermalize thanks to gauge interactions [11]. If minimally coupled scalars would not be present (i.e., $N_s=0$) the model would not be consistent since the Universe will be dominated by gravitons with (nonthermal) spectrum given by Eq. (2.3). The energy density of the quanta associated with the minimally coupled scalars, amplified thanks to the background transition from the inflationary phase to the stiff phase, will decrease with the Universe expansion as a^{-4} whereas the energy density of the background will decrease as a^{-6}. The moment at which the energy density of the background becomes subleading marks the beginning of the radiation dominated phase and it takes place at a (present) frequency of the order of the mHz [13]. Notice that this frequency has been obtained by requiring the reheating mechanism to be only gravitational [31]. This assumption might be relaxed by considering different reheating mechanisms [32] (see also Ref. [33]). In order to satisfy the nucleosynthesis constraint in the framework of a quintessential model with gravitational reheating [31] we have to demand that [11,13]

$$\frac{3}{N_s}\left(\frac{g_{\rm n}}{g_{\rm th}}\right)^{1/3}<0.07, \tag{2.6}$$

where the factor of 3 counts the two polarizations of the gravitons but also the quanta associated with the inflaton and $g_n=10.75$ is the number of spin degrees of freedom at t_n. For frequencies $f(\eta_0)>f_1(\eta_0)$ the spectra of the produced gravitons are exponentially suppressed as $\exp[-f/f_1]$. This is a general feature of the spectra of massless particles pro duced thanks to the pumping action of the background geometry [12].[3]

III. SIGNAL-TO-NOISE RATIO FOR MONOTONIC BLUE SPECTRA

In order to detect a stochastic gravitational wave background in an optimal way we have to correlate the outputs of two (or more) detectors [15–18]. The signal received by a single detector can be thought as the sum of two components: the *signal* (given by the stochastic background itself) and the *noise* associated with each detector's measurement. The noise level associated with a single detector is, in general, larger than the expected theoretical signal. This statement holds for most of the single (operating and/or foreseen) gravitational waves detectors (with the possible exception of the Laser Interferometer Space Antenna (LISA) space interferometer [6]). Suppose now that instead of a single detectors we have a *couple* of detectors or, ideally, *a network* of detectors. The signal registered at each detector will be

$$s_i = h_i(t) + n_i(t), \tag{3.1}$$

where the index i labels each different detector. If the detectors are sufficiently far apart the ensamble average of the Fourier components of the noises is stochastically distributed which means that

$$\langle n_i^*(f) n_j(f') \rangle = \frac{1}{2}\,\delta(f-f')\,S_n^{(i)}(|f|), \tag{3.2}$$

where $S_n(|f|)$ is the one-sided noise power spectrum which is usually expressed in seconds. The very same quantity can be defined for the signal. By then assuming the noise levels to be statistically independent of the gravitational strain registered by the detectors we obtain Eq. (1.2).

Consider now the case of two correlated interferometers and define the following rescaled quantities: $\Sigma_n^{(i)} = S_n^{(i)}/S_0$ $(i = 1,2)$; $\nu = f/f_0$; $\Omega_{\rm GW}(f) = \Omega(f_0)\,\omega(f)$. (In this section we will not write the explicit dependence of the theoretical quantities upon η_0: they are meant to be considered at the present time.) Notice that f_0 is (approximately) the frequency where the noise power spectra are minimal and $\Omega_{\rm GW}(f_0)$ is the graviton (logarithmic) energy density at the frequency f_0. Therefore the signal-to-noise ratio can be expressed as

$$\mathrm{SNR}^2 = \frac{3H_0^2}{5\sqrt{2}\,\pi^2}\,\sqrt{T}\,\frac{\Omega_{\rm GW}(f_0)}{f_0^{5/2}\,S_0}\,J, \tag{3.3}$$

where we defined the (dimension-less) integral

$$J^2 = \int_0^\infty d\nu\,\frac{\gamma^2(f_0\nu)\,\omega^2(f_0\nu)}{\nu^6\,\Sigma_n^{(1)}(f_0\nu)\,\Sigma_n^{(2)}(f_0\nu)}. \tag{3.4}$$

From this last expression we can deduce that the minimum detectable $h_0^2\,\Omega_{\rm GW}(f_0)$ is given by (1 yr $= \pi \times 10^7$ s)

$$h_0^2\,\Omega_{\rm GW}(f_0) \simeq 4.0\times 10^{32}\,\frac{f_0^{5/2} S_0}{J}\left(\frac{1\ \mathrm{yr}}{T}\right)^{1/2}\mathrm{SNR}^2. \tag{3.5}$$

For example, by taking $f_0 = 100$ Hz and $S_0 = 10^{-44}$ s, we get

$$h_0^2\,\Omega_{\rm GW}(100\,\mathrm{Hz}) \simeq \frac{4.0\times 10^{-7}}{J}\left(\frac{1\ \mathrm{yr}}{T}\right)^{1/2}\mathrm{SNR}^2. \tag{3.6}$$

Therefore, the estimate of the sensitivity of cross-correlation measurements between two detectors to a given spectrum $\Omega_{\rm GW}(f)$ reduces, in our case, to the calculation of the integral J defined in Eq. (3.4). Given a specific theoretical spectrum, J can be numerically determined for the wanted pair of detectors.

IV. ACHIEVABLE SENSITIVITIES FOR QUINTESSENTIAL SPECTRA

Consider first the case of the two LIGO detectors (located at Hanford, WA and Livingston, LA) in their ''advanced'' versions. From the knowledge of the geographical locations and orientations of these detectors [34], the overlap reduction function can be calculated [17,18], and the result is reported in Fig. 2 of Ref. [18]. As function of the frequency, γ has its first zero at 64 Hz and it falls rapidly at higher frequency. This behavior allows us to restrict the integration domain in Eq. (3.4) to the region $f \leqslant 10$ kHz (i.e., $\nu \leqslant 100$). We assumed identical construction of the two detectors (i.e., $S_n^{(1)} = S_n^{(2)}$). For the rescaled noise power spectrum of each detector we used the analytical fit of Ref. [35], namely (see Fig. 1),

$$\Sigma_n(f) = \begin{cases} \infty, & f < f_b, \\ h_a^2\left(\dfrac{f_a}{\Gamma}\right)^3 \dfrac{1}{f^4}, & f_b \leqslant f < \dfrac{f_a}{\Gamma}, \\ h_a^2, & \dfrac{f_a}{\Gamma} \leqslant f < \Gamma f_a, \\ \dfrac{h_a^2}{(\Gamma f_a)^3} f^2, & f \geqslant \Gamma f_a, \end{cases} \tag{4.1}$$

with

$$h_a^2 = 1.96\times 10^{-2} \quad \Gamma = 1.6 \quad f_a = 68\,\mathrm{Hz} \quad f_b = 10\,\mathrm{Hz}.$$

In the case of a flat spectrum [i.e., $\alpha = 0$, $\omega(f) = 1$] we find $J \simeq 6.1\times 10^3$, which implies

[3]Quintessential graviton spectra have, in general, three branches: a soft branch (for $10^{-18}\,\mathrm{Hz} \lesssim f \lesssim 10^{-16}\,\mathrm{Hz}$), a semihard branch (for $10^{-16}\,\mathrm{Hz} \lesssim f \lesssim 10^{-3}\,\mathrm{Hz}$) and a hard branch which is the one mainly discussed in the present paper. The reason for this choice is obvious since the noise power spectra of the interferometric detectors are defined in a band which falls in the region of the hard branch of the theoretical spectrum.

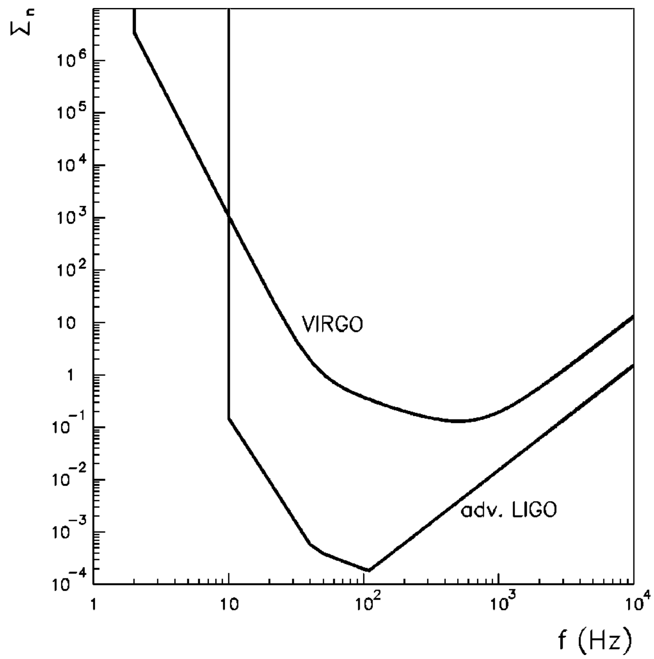

FIG. 1. We report the rescaled noise power spectra of the LIGO and VIRGO detectors used for the calculation of the signal-to-noise ratio.

$$h_0^2\,\Omega_{\rm GW}(100\,{\rm Hz}) \simeq 6.5\times 10^{-11}\left(\frac{1\ {\rm yr}}{T}\right)^{1/2}{\rm SNR}^2 \tag{4.2}$$

in close agreement with the estimate obtained in Ref. [18]. The minimum detectable $h_0^2\,\Omega_{\rm GW}$ for quintessential gravitons can be obtained by recalling that

$$\omega(f) = \frac{\nu}{\ln^2\nu_1}\ln^2\left(\frac{\nu}{\nu_1}\right).$$

For $f_0 = 100$ Hz, numerical integration gives

$$J \simeq \frac{10^3}{\ln^2\nu_1}\{6.91 + 21.36\ln\nu_1 + 26.52\ln^2\nu_1 + 15.68\ln^3\nu_1 + 3.78\ln^4\nu_1\}^{1/2},$$

or, taking into account Eq. (2.4), in terms of N_s:

$$J \simeq \frac{1.6\times 10^7}{(88.0-\ln N_s)^2}\,P_{\rm L}(N_s) \tag{4.3}$$

with

$$P_{\rm L}^2(N_s) \simeq 1.07 - 4.62\times 10^{-2}\ln N_s + 7.52\times 10^{-4}\ln^2 N_s - 5.44\times 10^{-6}\ln^3 N_s + 1.48\times 10^{-8}\ln^4 N_s\,.$$

By inserting this expression in Eq. (3.6), one has

$$h_0^2\,\Omega_{\rm GW}(100\,{\rm Hz}) \simeq 2.5\times 10^{-14}\frac{(88.0-\ln N_s)^2}{P_{\rm L}(N_s)}\left(\frac{1\ {\rm yr}}{T}\right)^{1/2}\times {\rm SNR}^2. \tag{4.4}$$

By assuming for N_s the minimum value compatible with Eq. (2.6) (i.e., $N_s = 21$), we obtain

$$h_0^2\,\Omega_{\rm GW}(100\,{\rm Hz}) \simeq 1.8\times 10^{-10}\left(\frac{1\ {\rm yr}}{T}\right)^{1/2}{\rm SNR}^2 \tag{4.5}$$

As we can see by comparing Eq. (4.2) with Eq. (4.5) the minimum detectable $h_0^2\,\Omega_{\rm GW}(100\,{\rm Hz})$ is slightly larger for growing spectra. This a general result that is simply related to the structure of J. For the special value of N_s considered this difference is roughly of a factor of 2. Another important point to stress is that for both the graviton spectra considered, as a consequence of the frequency behavior of $\gamma(f)$ and the presence of the weighing factor ν^{-6} in the integrand, the main contribution to the integral J comes from the region $f < 100$ Hz. The cutoff introduced by the overlap reduction function is not so relevant: by assuming $\gamma(f) = 1$ over the whole integration domain (i.e., considering the correlation of one of the detector with itself), the sensitivity increases only by a factor 2.4 in the case of a flat spectrum, and 3.6 in the case of the quintessential one. This means that the only way to get a substantial rise in sensitivity lies in the improvement of the noise characteristics of the detectors in the low-frequency region.

As a comparison we considered also the sensitivity that could be obtained at VIRGO in the (purely hypothetical) case in which the detector now under construction at Cascina, near Pisa (Italy), were correlated with a second interferometer located at about 50 km from the first and with the same orientation.[4] The overlap reduction function for this correlation has its first zero at a frequency $f \sim 3$ kHz (see Fig. 2).

Also in this case we assumed that the detectors are identical and for the common rescaled noise power spectrum we used the analytical parametrization given in Ref. [36] (see Fig. 1)

$$\Sigma_n(f) = \begin{cases} \infty, & f < f_b\,, \\ \Sigma_1\left(\frac{f_{\rm a}}{f}\right)^5 + \Sigma_2\left(\frac{f_{\rm a}}{f}\right) + \Sigma_3\left[1+\left(\frac{f}{f_{\rm a}}\right)^2\right], & f \geq f_b\,, \end{cases} \tag{4.6}$$

where

[4]For illustrative purposes, we assumed, within our example, 50 km as the minimum distance sufficient to decorrelate local seismic and e.m. noises. This hypothesis might be proven to be correct and it is certainly justified in the spirit of this exercise. However, at the moment, we do not have any indication either against or in favor of our assumption.

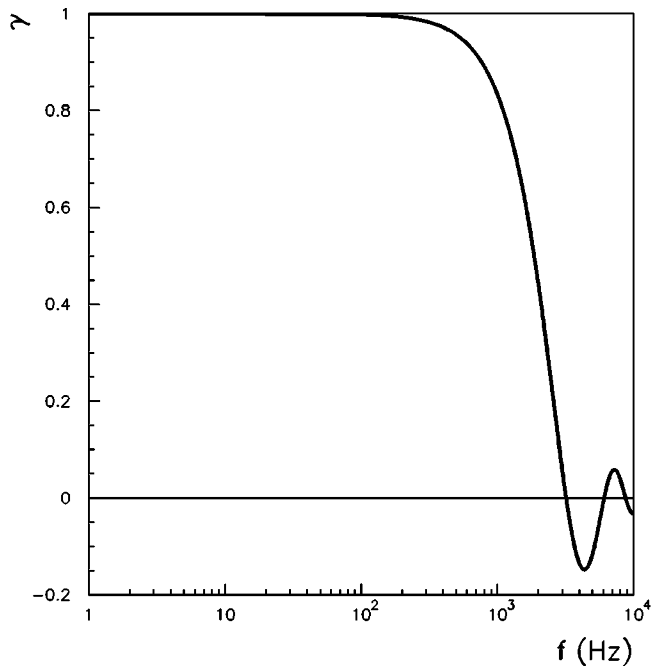

FIG. 2. The overlap reduction function for the correlation of VIRGO with a coaligned interferometer whose central (corner) station is located at (43.2 N, 10.9 E), $d \simeq 58.0$ km (Italy).

$$f_a = 500\ \text{Hz}, \quad f_b = 2\ \text{Hz}, \quad \begin{aligned} \Sigma_1 &= 3.46\times 10^{-6},\\ \Sigma_2 &= 6.60\times 10^{-2},\\ \Sigma_3 &= 3.24\times 10^{-2},\end{aligned}$$

In the case of flat spectrum, limiting the numerical integration to 10 kHz, we obtain $J \simeq 5.5$ and, therefore, according to Eq. (3.6)

$$h_0^2\,\Omega_{\rm GW}(100\ \text{Hz}) \simeq 7.2\times 10^{-8}\left(\frac{1\ \text{yr}}{T}\right)^{1/2}\text{SNR}^2. \quad (4.7)$$

In the case of quintessential inflation, for $f_0 = 100$ Hz, we have

$$J \simeq \frac{1}{\ln^2\nu_1}\{5.79 - 0.30\ln\nu_1 + 31.20\ln^2\nu_1 + 6.11\ln^3\nu_1 + 12.91\ln^4\nu_1\}^{1/2}$$

or, in terms of N_s,

$$J \simeq \frac{1.6\times 10^4}{(88.0 - \ln N_s)^2}\,P_{\rm V}(N_s) \quad (4.8)$$

with

$$P_{\rm V}^2(N_s) \simeq 3.10 - 0.14\ln N_s + 2.37\times 10^{-3}\ln^2 N_s - 17.84\times 10^{-6}\ln^3 N_s + 5.04\times 10^{-8}\ln^4 N_s.$$

Therefore, from Eq. (3.6) one has

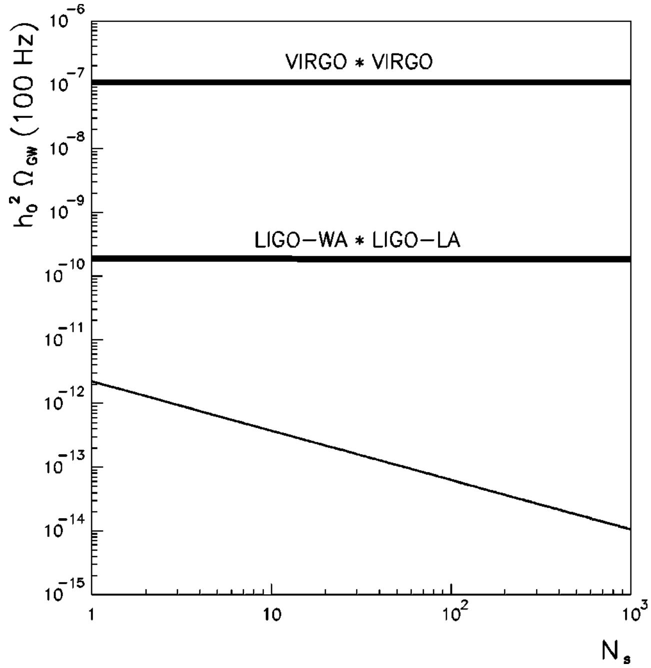

FIG. 3. We report the theoretical amplitude computed in Eq. (4.11) (full thin line) and the associated sensitivities computed in Eqs. (4.4) and (4.9) for $T = 1$ yr and SNR = 1 (full thick lines).

$$h_0^2\,\Omega_{\rm GW}(100\ \text{Hz}) \simeq 2.5\times 10^{-11}\frac{(88.0 - \ln N_s)^2}{P_{\rm V}(N_s)}\left(\frac{1\ \text{yr}}{T}\right)^{1/2}\text{SNR}^2 \quad (4.9)$$

that for $N_s = 21$ gives

$$h_0^2\,\Omega_{\rm GW}(100\,\text{Hz}) \simeq 1.1\times 10^{-7}\left(\frac{1\ \text{yr}}{T}\right)^{1/2}\text{SNR}^2. \quad (4.10)$$

At a frequency of 100 Hz the theoretical signal can be expressed as

$$h_0^2\,\Omega_{\rm GW}(100\,\text{Hz}) = N_s^{-3/4}\times 10^{-15}[2220.07 - 50.46\ln N_s + 0.28\ln^2 N_s], \quad (4.11)$$

as a function of N_s. In Fig. 3 this function (full thin line) is compared with the sensitivity of LIGO-WA*LIGO-LA and VIRGO*VIRGO (full thick lines) obtained from, respectively, Eqs. (4.4) and (4.9), assuming $T = 1$ yr and SNR = 1. We can clearly see that our signal is always below the achievable sensitivities. Notice that, if we assume purely gravitational reheating $N_s \gtrsim 21$.

One could think that, thanks to the sharp growth of the spectrum, the signal could be strong enough around 10 kHz, namely at the extreme border of the interferometers band. Indeed around $f_0 = 10$ kHz, the theoretical signal is given by

$$h_0^2\,\Omega_{\rm GW}(10\ \text{kHz}) = N_s^{-3/4}\times 10^{-15}[1387.81 - 39.89\ln N_s + 0.28\ln^2 N_s]. \quad (4.12)$$

We see that the situation does not change qualitatively. In fact it is certainly true that around 10 kHz the signal is larger but the sensitivity is also smaller. In fact, repeating the calculation for $f_0 = 10$ kHz, in the case $N_s = 21$ we obtain

$$h_0^2\,\Omega_{\rm GW}(10\,{\rm kHz}) \simeq \begin{cases} 1.1\times 10^{-8}\left(\dfrac{1\ {\rm yr}}{T}\right)^{1/2}\ {\rm SNR}^2, \\ \text{LIGO-WA*LIGO-LA}, \\ 6.7\times 10^{-6}\left(\dfrac{1\ {\rm yr}}{T}\right)^{1/2}\ {\rm SNR}^2, \\ \text{VIRGO*VIRGO}. \end{cases} \tag{4.13}$$

If we compare Eqs. (4.13) with Eqs. (4.5) and (4.10) we see that the minimum detectable signal gets larger the larger is the spectral frequency. Therefore, the mismatch apparent from Fig. 3 between the theoretical signal and the experimental sensitivity will remain practically unchanged.

V. CONCLUDING REMARKS

In this paper we precisely computed the sensitivity of pairs of interferometric detectors to blue and mildly violet spectra of relic gravitons. Our investigation can be of general relevance for any model predicting non flat spectra of relic gravitons. We analyzed the correlation of the two Laser Interferometer Gravitational Wave Observatory (LIGO) detectors in their ''advanced'' phase. On a more speculative ground we investigated the theoretical possibility of the correlation of VIRGO with an identical, coaligned, interferometer located very near to it.

As a test for our techniques we first discussed the case of a flat spectrum which has been discussed in the past. We then applied our results to the case of quintessential inflationary models whose graviton spectra are, in general, characterized by three ''branches.'' A soft branch (in the far infrared of the graviton spectrum around 10^{-18}–10^{-16} Hz), a semihard branch (between 10^{-16} and 10^{-3} Hz) and a truly hard branch ranging, approximately, from 10^{-3} Hz to 100 GHz. Since the interferometers band is located, roughly, between few Hz and 10 kHz, the relevant signal will come from the hard branch of the spectrum whose associated energy density appears in the signal-to-noise ratio with blue (or mildly violet) slope. In the hard branch the energy density of quintessential gravitons is maximal for frequencies in the range of the GHz. In this region $h_0^2\,\Omega_{\rm GW}$ can be as large as 10^{-6}. In spite of the fact quintessential spectra are growing in frequency the predicted signal is still too small and below the sensitivity achievable by the advanced LIGO detectors. The reason for the smallness of the signal in the region $f \sim 1$ kHz is twofold. On one hand we have to enforce the nucleosynthesis bound on the spectrum. On the other hand, because of the gravitational reheating mechanism adopted, the number of (minimally coupled) scalar degrees of freedom needs to be large. It might be possible, in principle, that different reheating mechanisms could change the signal for frequencies comparable with the window of the interferometers. Therefore, the analysis presented in this paper seems to suggest that new techniques (possibly based on electromagnetic detectors [37]) operating in the GHz region should be used in order to directly detect quintessential gravitons. The first feasibility studies of electromagnetic detectors applied to relic gravitons have indeed been presented in the seventies [19]. In light of the present technological capabilities those studies should be again considered. We hope to came back on this issue in a future publication.

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