

## NOTE ON THE UNILATERAL SURFACE OF MOEBIUS \*

BY

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In order to construct an algebraic surface containing as a part the unilateral paper-strip of MOEBIUS,† let a straight line  $L$  move in space along a circle  $C$ , perpendicular to the tangents of  $C$  and in such a way that, when the point of intersection  $Q$  of  $L$  with  $C$  has described the full circle, the initial position of  $L$  makes with its final position an angle of  $180^\circ$ . The condition that  $L$  meets  $C$  at right angles is equivalent to the condition that  $L$  meets a straight line  $A$  passing through the center  $M$  of the circle and perpendicular to its plane; let  $P$  be the movable point of intersection of  $L$  and  $A$ . If now we add the further condition that the range  $P$  on  $A$  be projective to the range  $Q$  on  $C$  (e. g., by taking the angle  $QPM$  always half the angle of the arc described by  $Q$  on  $C$ ) then  $L$  describes, according to a general theorem,‡ a *ruled surface of the third order*.

Conversely: take any ruled surface  $R$  of the third order, particular cases excepted, pass a plane section through one of the generators  $L$  which will meet  $R$  besides  $L$  in a conic section  $K$ , and describe a curve  $T$  on  $R$  the points of which have along the generators a sufficiently small constant distance from  $K$ ; then  $T$  will cut out of  $R$  a unilateral Moebius surface.

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† MOEBIUS: *Ueber die Bestimmung des Inhalts eines Polyeders*, § 11. (Gesammelte Werke, vol. II, p. 484-485.)

‡ REYE: *Geometrie der Lage*, ed. 3, vol. I, p. 209, no. 101.