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ERRATA TO "CONCERNING ARCWISE CONNECTEDNESS AND THE EXISTENCE OF SIMPLE CLOSED CURVES IN PLANE CONTINUA"

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Substitute the following for the last two paragraphs in the proof of Theorem 2 of my paper [1].

Assume without loss of generality that the sequence $F_1, F_2, F_3, \ldots$ is such that for each positive integer n, there exist two arc-segments R_n and E_n such that (1) $R_n \subset \text{Bd}\ R$, (2) $E_n \subset \text{Bd}\ E$, and (3) each arc-segment meets $F_1, F_2, F_3, \ldots$ only in F_{2n} and has one endpoint in F_{2n-1} and the other endpoint in F_{2n+1}. Let $p_1, p_2, p_3, \ldots$ be a sequence of points converging to p such that for each positive integer n, the point p_n is in $F_{2n} \cap (R - \text{Cl}\ E)$. The sequence $R_1, R_2, R_3, \ldots$ converges to a point w_1 of $M \cap \text{Bd}\ R$ and $E_1, E_2, E_3, \ldots$ converges to a point w_2 of $M \cap \text{Bd}\ E$.

Since M is aposyndetic, for $n=1$ and 2, there exist a subcontinuum M_n of M and a circular region G_n ($\text{Cl}\,(G_1 \cup G_2) \cap \{y, z\} = \text{Cl}\ G_1 \cap \text{Cl}\ G_2 = \varnothing$) in S such that G_n contains w_n and meets only one component of $\text{Bd}\,(R-E)$, the point p is in the interior of M_n relative to M and $\text{Cl}\ G_n \cap M_n = \varnothing$. Let G denote a circular region in S containing p such that $\text{Cl}\ G \cap \text{Cl}\,(G_1 \cup G_2) = \varnothing$ and $G \cap M$ is contained in $M_1 \cap M_2$. Assume without loss of generality that for each positive integer i, $\text{Cl}\ R_i \subset G_1$, $\text{Cl}\ E_i \subset G_2$, and p_i belongs to G. Let k be a positive integer such that $\text{Cl}\,(U_k \cup V_k) \cap \text{Cl}\,(G_1 \cup G_2) = \varnothing$ and $F_1, F_2, \ldots, F_7$ all lie in Y_k. Let j be a positive integer such that $U_k \cup V_k \supset \text{Cl}\,(U_j \cup V_j)$. Let P_1 be a circular region in G centered on p_1 such that $\text{Cl}\ P_1$ does not meet $F_1 \cup F_3 \cup R_1 \cup E_1$. Since M is not aposyndetic at p_1 with respect to $\{y, z\}$, the component of $M-(U_j \cup V_j)$ which contains p_1 is not open relative to M at p_1. Hence the boundary of P_1 contains an arc-segment S_1 whose endpoints a_1 and b_1 lie in different components of $M-(U_j \cup V_j)$ such that $M \cap S_1 = \varnothing$. There exists a simple closed curve C_1 which separates a_1 from b_1 in S and contains no point of $M-(U_j \cup V_j)$ such that $C_1 \cap S_1$ is connected. In C_1 there exists an arc-segment T_1 which crosses S_1, contains no point of $M \cup \text{Cl}\,(U_j \cup V_j)$, and has its endpoints in $\text{Bd}\,(U_j \cup V_j)$. Let P_2 be a circular region in G centered on p_2 such that $\text{Cl}\ P_2$ does not meet $F_3 \cup F_5 \cup R_2 \cup E_2 \cup T_1$. The component of $(M \cup S_1 \cup \text{Cl}\ T_1)-(U_j \cup V_j)$ which contains p_2 is not open relative to $M \cup S_1 \cup \text{Cl}\ T_1$ at p_2. Hence the boundary of P_2 contains

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an arc-segment S_2 whose endpoints a_2 and b_2 lie in different components of $(M \cup S_1 \cup \mathrm{Cl}\, T_1)-(U_j \cup V_j)$ such that $M \cap S_2=\varnothing$. There exists a simple closed curve C_2 which separates a_2 from b_2 in S and contains no point of $(M \cup S_1 \cup \mathrm{Cl}\, T_1)-(U_j \cup V_j)$ such that $C_2 \cap S_2$ is connected. In C_2 there exists an arc T_2 which crosses S_2, contains no point of $M \cup \mathrm{Cl}\,(U_j \cup V_j)$, and has its endpoints in $\mathrm{Bd}\,(U_j \cup V_j)$. Continue this process. There exist a circular region P_3 centered on p_3, arc-segments S_3 and T_3, and a simple closed curve C_3 such that (1) $\mathrm{Cl}\, P_3$ does not meet $F_5 \cup F_7 \cup R_3 \cup E_3 \cup T_1 \cup T_2$, (2) S_3 has endpoints a_3 and b_3 in M and is contained in $(S-M) \cap \mathrm{Bd}\, P_3$, (3) C_3 separates a_3 from b_3 and contains no point of $(M \cup \bigcup_{i=1}^{2} (S_i \cup \mathrm{Cl}\, T_i))-(U_j \cup V_j)$, (4) $C_3 \cap S_3$ is connected, and (5) T_3 is contained in $C_3-\mathrm{Cl}\,(U_j \cup V_j)$, meets S_3, and has its endpoints in $\mathrm{Bd}\,(U_j \cup V_j)$.

For each i ($i=1$, 2, and 3), no component of $\mathrm{Cl}\,(G_1 \cup G_2)$ contains both a point of T_i which precedes and a point of T_i which follows $T_i \cap S_i$ with respect to the order of T_i; for otherwise, T_i union a component of $\mathrm{Bd}\,(G_1 \cup G_2)$ would separate a_i from b_i in S which contradicts the existence of M_1 and M_2 [5, Theorem 32, p. 181]. For $m=1$, 2, and 3, let v_m denote a point of $\mathrm{Bd}\,(U_j \cup V_j) \cap \mathrm{Cl}\, T_m$. For $m=1$, 2, and 3, since $F_{2m-1} \cup F_{2m+1} \cup R_m \cup E_m$ separates v_m from $T_m \cap S_m$ in S and $F_{2m-1} \cup F_{2m+1}$ is in Y_k, there exists a component L_m of $R_m \cup E_m$ such that $L_m \cup Y_k$ separates v_m from $T_m \cap S_m$ in S [5, Theorem 20, p. 173]. For $m=1$, 2, and 3, the set $L_m \cup Y_k$ separates v_m from the other endpoint z_m of T_m in S. To see this suppose that for some $m=1$, 2, or 3, $L_m \cup Y_k$ does not separate v_m from z_m in S. It follows that $T_m \cap L_m$ contains a point which precedes and a point which follows $T_m \cap S_m$ with respect to the order of T_m. Since L_m is contained in a component of $G_1 \cup G_2$, this is impossible. Note that for $m=1$, 2, and 3, z_m does not belong to the v_m-component of $\mathrm{Bd}\,(U_j \cup V_j)$. Assume without loss of generality that for $m=1$, 2, and 3, the point v_m belongs to $\mathrm{Bd}\, U_j$.

Some two of L_1, L_2, and L_3, say L_1 and L_2, are contained in the same component of $G_1 \cup G_2$. Assume that $L_1 \cup L_2$ is contained in G_1. For $m=1$ and 2, let D_m be the arc-segment $(R_m \cup E_m)-L_m$. Let A be an arc in $G_2 \cap \mathrm{Bd}\, E$ which contains $D_1 \cup D_2$. There exists an arc-segment B in $E \cap G_2$ such that $\mathrm{Cl}\, B$ contains the endpoints of A. The simple closed curve $A \cup B$ contains $D_1 \cup D_2$ and there exists a complementary domain K of $A \cup B$ which does not meet $F_1 \cup F_3 \cup F_5 \cup \mathrm{Bd}\, R$. Assume that for $m=1$ and 2, the order of the arc-segment T_m indicates that T_m goes from v_m to z_m. For $m=1$ and 2, since $L_m \cup Y_k$ separates v_m from $T_m \cap S_m$, the arc-segment T_m must meet L_m before S_m; and since $L_m \cup D_m \cup F_{2m-1} \cup F_{2m+1}$ separates $S_m \cap T_m$ from z_m and $L_m \cap T_m$ does not contain both a point which precedes and a point which follows $T_m \cap S_m$, $T_m \cap D_m$ must contain a point which follows $T_m \cap S_m$ with respect to the order of T_m. If for $m=1$ or 2, $T_m \cap (A \cup B)$ contains a point which precedes and a point which follows $T_m \cap S_m$ with respect to the order of T_m, then $T_m \cup A \cup B$ separates a_m from b_m in S and does not meet M_2. This contradicts the fact that M_2 is connected. Hence for $m=1$ and 2, T_m must meet $A \cup B$ only at points which follow $T_m \cap S_m$ with respect to the order of T_m.

For $m=1$ and 2, let c_m be the first point of $T_m \cap (A \cup B)$ with respect to the order of T_m, let K_m be the arc-segment in T_m from v_m to c_m, and let N_m denote the arc in Cl T_m which goes from c_m to z_m. Note that for $m=1$ and 2, the point c_m is in D_m. The set $K_1 \cup K_2 \cup \text{Bd } U_j$ is contained in $S-\text{Cl } K$. For $m=1$ and 2, since T_m does not meet Bd $(U_j \cup V_j)$, $N_m \cap \text{Bd } U_j = K_m \cap \text{Bd } V_j = \varnothing$. Let U be an arc-segment in $K_1 \cup K_2 \cup \text{Bd } U_j$ which has endpoints c_1 and c_2. The arc Cl U separates F_1 from F_3 in $S-K$ [5, Theorem 28, p. 156]. Hence there exist arcs X_1 and X_2 in Cl L_1 such that (1) X_1 and X_2 abut on Cl U from opposite sides with respect to a simple closed curve in $U \cup A \cup B$, (2) $X_1 \cap F_1 \neq \varnothing$, and (3) $X_2 \cap F_3 \neq \varnothing$. Since $\text{Cl } T_1 \cap \text{Cl } T_2 = \varnothing$, $(N_1 \cup N_2) \cap \text{Cl } (U_j \cup G_1) = \varnothing$, and $(K_1 \cup K_2) \cap \text{Bd } V_j = \varnothing$, there exists a simple closed curve J in $U \cup N_1 \cup N_2 \cup \text{Bd } V_j$ such that X_1 and X_2 abut on Cl U from opposite sides with respect to J. It follows that J separates F_1 from F_3 in S. Since $J \cap (M-(U_k \cup V_k)) = \varnothing$ and $F_1 \cup F_3$ is contained in the x-component of $M-(U_k \cup V_k)$, this is a contradiction. Evidently $L_1 \cup L_2$ is not contained in G_1. It can be shown by the same method that assuming $L_1 \cup L_2$ is in G_2 also involves a contradiction. It follows that M is not aposyndetic at p with respect to both w_1 and w_2 which contradicts the hypothesis of the theorem. Hence L^x_{yz} is locally connected.

References

1. Charles L. Hagopian, *Concerning arcwise connectedness and the existence of simple closed curves in plane continua*, Trans. Amer. Math. Soc. **147** (1970), 389–402.

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