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ERRATUM TO "GENERALIZED SUPER-SOLUTIONS OF PARABOLIC EQUATIONS"

BY

NEIL EKLUND

It has been brought to my attention that two terms were not defined correctly. The necessary changes will be made in Definitions 2, 3, and 6 and this will require a slight modification in the statement of Theorem 3. However, the content of Theorem 3 will not be changed.

DEFINITION 2. $u = u(x, t)$ is called a *super-solution* of $Lu = 0$ in Q if $u \in L^2[0, T; H^{1,2}(\Omega)]$ and, for all $v \in C_0^1(Q^i)$ with $v \geqslant 0$,

$$\iint_Q \left[a_{ij}u,_i v,_j + d_j v,_j u - b_j u,_j v - cuv - uv_t \right] dx\, dt \geqslant 0. \tag{7}$$

DEFINITION 3. Let $u \in L^2[0, T; H^{1,2}(\Omega)]$.

(a) u is *nonnegative* on $\partial\Omega \times [0, T]$ if there is a sequence of functions $\{u^k(x, t)\} \subset C(\overline{Q})$ with u^k Lipschitz continuous in x on $\overline{\Omega}$ uniformly in t, $u^k > 0$ on $\partial\Omega \times [0, T]$, and $u^k \to u$ in $L^2[0, T; H^{1,2}(\Omega)]$.

(b) u is *nonnegative* on $\Omega \times (0)$ if, for each function $v \in C_0(\Omega \times [0, T))$,

$$\liminf_{t \to 0} \int_\Omega u(x, t)v(x, t)\, dx \geqslant 0.$$

In particular, if u has initial values $u_0(x) \in L^2(\Omega)$ on $\Omega \times (0)$, then u is *nonnegative* on $\Omega \times (0)$ if $\int_\Omega u_0(x)v(x)\, dx \geqslant 0$ for all $v \in C_0(\Omega)$ with $v \geqslant 0$.

(c) u is *nonnegative* on $\partial_p Q$ if u is nonnegative on $\partial\Omega \times [0, T]$ and on $\Omega \times (0)$.

Then Theorem 3 should be restated as follows:

THEOREM 3 (MINIMUM PRINCIPLE). *Let u be a super-solution of $Lu = 0$ in Q and assume $u \geqslant 0$ on $\partial_p Q$ with initial values $u_0 \in L^2(\Omega)$. Then $u \geqslant 0$ a.e. in Q.*

The proof of Theorem 3 is as stated in the paper. We now define $\mathcal{S}_Q$.

DEFINITION 6. (i) $\mathcal{S}_Q = \{u$ lower semicontinuous super-solution on $Q\}$.

(ii) $\mathcal{S}'_Q = L_Q \cap \{u; u$ is SMV$\}$.

(iii) $\mathcal{S}''_Q = L_Q \cap \{u;$ for each cylinder W, $\overline{W} \subset Q$, and $v \leqslant u$ on $\partial_p W$, $v \leqslant u$ on $W\}$.

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(iv) $\mathcal{S}_Q''' = L_Q \cap \{u;\ u \text{ is LSMV}\}$.

It may be of interest to the reader to state the counterexample to Theorem 3 using Definition 3 in the original article. It was sent to me by Dr. Filippo Chiarenza of the University of Catania, Italy. Consider $Lu = u_t - u_{xx}$ in $Q = (-1, 1) \times (0, 1)$ and let $u(x, t) = x^2 + 2t - 1$. Then $Lu = 0$ in Q. Now let $\{u^k(x, t)\}$ be defined by

$$u^k(x, t) = \begin{cases} kt(x^2 + 2t - 1) + 1/k, & 0 \leq t < 1/k, \\ x^2 + 2t - 1 + 1/k, & 1/k \leq t \leq 1. \end{cases}$$

Then $u^k > 0$ on $\partial_p Q$, $u^k \in C(\overline{Q})$, and u^k is lipschitz continuous in x on Ω uniformly in t. Since $u^k \to u$ in $L^2[0, T; H^{1,2}(\Omega)]$ we should have $u \geq 0$ on Q. Since $u(x, t) = x^2 + 2t - 1 < 0$ in a neighborhood of $(0, 0)$, we have the desired counterexample.

References

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Department of Mathematics, Centre College of Kentucky, Danville, Kentucky 40422