TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 253, September 1979

DIXMIER'S REPRESENTATION THEOREM OF CENTRAL DOUBLE CENTRALIZERS ON BANACH ALGEBRAS

BY

SIN-EI TAKAHASI

ABSTRACT. The present paper is devoted to a representation theorem of central double centralizers on a complex Banach algebra with a bounded approximate identity. In particular, our result implies the representation theorem of the ideal center of an arbitrary C^*-algebra established by J. Dixmier.

1. Introduction. R. C. Busby [2] has noted that every central double centralizer on any C^*-algebra can be represented as a bounded continuous complex-valued function on its structure space, which is equivalent to Dixmier's representation theorem [6, Theorem 5]. Let A be a complex Banach algebra with a bounded approximate identity and Prim A the structure space of A. A central double centralizer T on A may be identified with a bounded linear operator T on A such that $(Tx)y = x(Ty)$ for all $x, y \in A$. In this paper, we show that, if the ideal center of A has a Hausdorff structure space, every central double centralizer T on A can be represented as a bounded continuous complex-valued function Φ_T on Prim A such that $Tx + P = \Phi_T(P)(x + P)$ for all $x \in A$ and $P \in \text{Prim } A$. Here $x + P$ for $P \in \text{Prim } A$ denotes the canonical image of x in A/P. In particular, if A is a C^*-algebra then our representation theorem implies the Busby's or, equivalently, Dixmier's. In this way we get a proof of Dixmier's theorem, quite different from that given in [6]. This was inspired by Davenport's representation theorem of multiplier algebras on Banach algebras with bounded approximate identity [5, Theorem 2.8]. We also obtain a similar representation theorem of central double centralizers on a quasi-central Banach algebra with a completely regular center.

2. Davenport's representation theorem of $Z(M(A))$. In this paper, a complex Banach algebra with a bounded approximate identity will be denoted by A and the central double centralizer-algebra on A will be denoted by $Z(M(A))$, that is the center of the double centralizer-algebra $M(A)$ on A. Let $\{e_\alpha\}$ be the approximate identity on A and $A^{\#}$ the set of all elements f in the dual space A^* of A such that $\lim_\alpha \|f \cdot e_\alpha - f\| = 0$, where $f \cdot a(x) = f(ax)$ for

Received by the editors March 8, 1978 and, in revised form, July 6, 1978.
AMS (MOS) subject classifications (1970). Primary 46H15, 46J10; Secondary 46L20, 46L05.

0002-9947/79/0000-0408/$03.00

each a, $x \in A$ and $f \in A^*$. The set $A^\#$ is a closed subspace of A^* and $A^\# = \{f \cdot a : f \in A^*, a \in A\}$ (cf. [3], [5]). Then the dual space $(A^\#)^*$ of $A^\#$ becomes a Banach algebra under the restriction to $A^\#$ of the Arens product on the second dual space A^{**} of A. In fact, the restriction to $A^\#$ of the Arens product on A^{**} can be described as follows:

$$[G, f](a) = G(f \cdot a), \qquad F \cdot G(f) = F[G, f]$$

for each $a \in A, f \in A^\#$ and $F, G \in (A^\#)^*$. Let π be the canonical embedding of A into A^{**} and i the inclusion map of $A^\#$ into A^*. Put $\tau = i^*\pi$, where i^* denotes the dual map induced by i. Then the map τ is a norm reducing isomorphism of A into $(A^\#)^*$ [5, Lemma 2.5]. Furthermore, we can easily verify that $\|x\| \leqslant M\|\tau(x)\|$ for each $x \in A$, where M is the bound on $\{e_\alpha\}$. Then $\tau(A)$ is uniformly closed in $(A^\#)^*$. We now define $D(A)$ to be the set

$$D(A) = \{F \in (A^\#)^* : F \cdot \tau(A) + \tau(A) \cdot F \subset \tau(A)\}.$$

Then $D(A)$ is a Banach subalgebra of $(A^\#)^*$ and $\tau(A)$ is a closed two-sided ideal of $D(A)$.

DEFINITION 2.1. Let $Z(D(A))$ be the center of $D(A)$. The algebra $Z(D(A))$ is said to be the ideal center of A.

By Lemma 2.7 in [5], the algebra $(A^\#)^*$ has an identity J. If A is a C^*-algebra, then $A^\# = A^*$ and so $(A^\#)^* = A^{**}$ (cf. [5, Proof of Corollary 2.10]). Then $Z(D(A))$ becomes the ideal center of A in the sense of Dixmier [6].

J. Davenport has proved that there exists a continuous, algebraic isomorphism μ of $Z(M(A))$ into $(A^\#)^*$ such that $\tau(Tx) = (\mu T) \cdot \tau x = \tau x \cdot (\mu T)$ for all $x \in A$ and $T \in Z(M(A))$ [5, Theorem 2.8]. In fact, the map μ is given by

$$\mu T = \text{weak}^*\text{-}\lim_\alpha \tau(Te_\alpha)$$

for each $T \in Z(M(A))$. We then, from this Davenport result, obtain the following

LEMMA 2.2. *Let A be a Banach algebra with a bounded approximate identity $\{e_\alpha\}$ and μ the isomorphism of $Z(M(A))$ into $(A^\#)^*$ given by Davenport. Then $\mu(Z(M(A))) = Z(D(A))$.*

PROOF. Let $T \in Z(M(A))$. Since $\tau(A) \cdot (\mu T) = (\mu T) \cdot \tau(A) = \tau(T(A)) \subset \tau(A)$, we have $\mu T \in D(A)$. It is well known that $\pi(A)$ is weak*-dense in A^{**}. Moreover, i^* is a weak*-continuous map of A^{**} onto $(A^\#)^*$. These facts imply that $\tau(A)$ is weak*-dense in $(A^\#)^*$. Then for each $F \in (A^\#)^*$, there exists a net $\{x_\lambda\}$ in A such that $F = \text{weak}^*\text{-}\lim_\lambda \tau(x_\lambda)$. Let T^* be the dual map induced by T. Note that $T^*(f \cdot a) = f \cdot (Ta)$ for each $a \in A$ and $f \in A^*$, so that $T^*(A^{\#}) \subset A^{\#}$. Put $T^{\#} = T^*|A^{\#}$. Then we have

$$\begin{aligned}\lim_{\lambda} \tau(Tx_\lambda)(f) &= \lim_{\lambda} \tau(x_\lambda)(T^{\#}f) = F(T^{\#}f) \\ &= \lim_{\alpha} F\big(T^{\#}(f \cdot e_\alpha)\big) = \lim_{\alpha} \tau(Te_\alpha)[F, f] \\ &= (\mu T)[F, f] = (\mu T) \cdot F(f)\end{aligned}$$

for all $f \in A^{\#}$. On the other hand, we have

$$\begin{aligned}\lim_{\lambda} \tau(Tx_\lambda)(f) &= \lim_{\lambda} \tau(x_\lambda) \cdot (\mu T)(f) \\ &= \lim_{\lambda} \tau(x_\lambda)[\mu T, f] = F[\mu T, f] = F \cdot (\mu T)(f)\end{aligned}$$

for all $f \in A^{\#}$. We thus obtain $(\mu T) \cdot F = F \cdot (\mu T)$, so that $\mu T \in Z(D(A))$. Conversely, let $G \in Z(D(A))$. Put $\hat{G}(x) = \tau^{-1}(G \cdot \tau x)$ for each $x \in A$. Then $\hat{G}$ is a bounded linear operator on A. Furthermore, $\hat{G} \in Z(M(A))$. In fact, for each $x, y \in A$, $\tau((\hat{G}x)y) = \tau(\tau^{-1}(G \cdot \tau x)y) = (G \cdot \tau x) \cdot \tau y = \tau x \cdot (G \cdot \tau y)$, so that $(\hat{G}x)y = x\tau^{-1}(G \cdot \tau y) = x(\hat{G}y)$. We also have

$$\begin{aligned}\mu\hat{G}(f) &= \lim_{\alpha} \tau\big(\hat{G}e_\alpha\big)(f) = \lim_{\alpha} G \cdot \tau e_\alpha(f) \\ &= \lim_{\alpha} \tau e_\alpha \cdot G(f) = \lim_{\alpha} [G, f](e_\alpha) \\ &= \lim_{\alpha} G(f \cdot e_\alpha) = G(f)\end{aligned}$$

for all $f \in A^{\#}$. We thus get $\mu\hat{G} = G$ and our result is proved.

Furthermore, the following stronger result can be proved, and it is similar to one established by K. Saito [7], in which he has given a characterization of double centralizer-algebras on Banach algebras under some conditions. This was pointed out by the referee.

PROPOSITION 2.3. *Let A be a Banach algebra with a bounded approximate identity $\{e_\alpha : \alpha \in \Lambda\}$. Then there exists a continuous, algebraic isomorphism ν of the double centralizer-algebra $M(A)$ on A onto $D(A)$ which is an extension of μ to $M(A)$.*

PROOF. Let $(T, S) \in M(A)$. Since $\{\tau(Te_\alpha) : \alpha \in \Lambda\}$ is bounded, it has a weak*-convergent subnet $\{\tau(Te_{\alpha'}) : \alpha' \in \Lambda'\}$ in $(A^{\#})^*$. We now define the map ν from $M(A)$ to $(A^{\#})^*$ by

$$\nu(T, S) = \text{weak}^*\text{-}\lim_{\alpha'} \tau(Te_{\alpha'})$$

for each $(T, S) \in M(A)$. Then ν is well defined. In fact, let $\{\tau(Te_{\alpha'}) : \alpha' \in \Lambda'\}$ converge to F and $\{\tau(Te_{\alpha''}) : \alpha'' \in \Lambda''\}$ converge to G, each in the weak*-topology. Then for each $x \in A$ and $f \in A^{\#}$, we have

$$\begin{aligned}F \cdot \tau x(f) &= \lim_{\alpha'} \tau(Te_{\alpha'}) \cdot \tau x(f) \quad \text{(from [5, 2.6.1])} \\ &= \lim_{\alpha'} \tau(T(e_{\alpha'}x))(f) = \lim_{\alpha'} f(T(e_{\alpha'}x)) \\ &= f(Tx) = \tau(Tx)(f).\end{aligned}$$

Similarly, $G \cdot \tau x(f) = \tau(Tx)(f)$ for each $x \in A$ and $f \in A^{\#}$. Hence $(F - G) \cdot \tau x = 0$ for each $x \in A$. It follows by [5, 2.6.3] that $F = G$. Clearly, ν is linear. Also, since $\nu(T, S) \cdot \tau x = \tau(Tx)$ for each $x \in A$ as was seen in the above argument, we have $\nu(T, S) \cdot \tau(A) \subset \tau(A)$. Furthermore, we have

$$\begin{aligned} \tau x \cdot \nu(T, S)(f) &= \lim_{\alpha} \tau x \cdot \tau(Te_{\alpha})(f) \quad (\text{from } [5, 2.6.2]) \\ &= \lim_{\alpha} \tau((Sx)e_{\alpha})(f) = \lim_{\alpha} f((Sx)e_{\alpha}) \\ &= f(Sx) = \tau(Sx)(f) \end{aligned}$$

for each $x \in A$ and $f \in A^{\#}$. Then $\tau x \cdot \nu(T, S) = \tau(Sx)$ for each $x \in A$, so that $\tau(A) \cdot \nu(T, S) \subset \tau(A)$. In other words, $\nu(M(A)) \subset D(A)$. Now let (T, S), $(T', S') \in M(A)$. Set $F = \nu(T, S)$, $F' = \nu(T', S')$ and $F'' = \nu(TT', S'S)$. Then $F'' \cdot \tau x = \tau(TT'x)$ for each $x \in A$. On the other hand, for each $x \in A$, $F \cdot F' \cdot \tau x = F \cdot \tau(T'x) = \tau(TT'x)$, and hence $(F'' - F \cdot F') \cdot \tau x = 0$. It follows by [5, 2.6.3] that $F'' = F \cdot F'$. In other words, $\nu((T, S)(T', S')) = \nu(T, S) \cdot \nu(T', S')$. If $F = F'$, then $\tau((T - T')x) = (F - F') \cdot \tau x = 0$ for each $x \in A$, and so $T = T'$ (and hence $S = S'$) since τ is one-to-one. Thus ν is an algebraic isomorphism of $M(A)$ into $D(A)$. To show that ν is onto, let $F \in D(A)$ and set

$$L_F(x) = \tau^{-1}(F \cdot \tau x), \qquad R_F(x) = \tau^{-1}(\tau x \cdot F)$$

for each $x \in A$. Then (L_F, R_F) is an element of $M(A)$. We further have

$$\left(\nu(L_F, R_F) - F\right) \cdot \tau x = \tau(L_F x) - F \cdot \tau x = 0$$

for each $x \in A$. It follows that $\nu(L_F, R_F) = F$. Thus ν is onto. Let $(T, S) \in M(A)$. Since $\nu(T, S) = \text{weak*-lim}_{\alpha} \tau(Te_{\alpha})$ and $\|\tau(Te_{\alpha})\| \leqslant \|Te_{\alpha}\| \leqslant M\|(T, S)\|$, it follows that $\|\nu(T, S)\| \leqslant M\|(T, S)\|$.

Finally we can easily see that the restriction to $Z(M(A))$ of ν is equal to μ from the definition of ν.

Remark 2.4. Let A be as in Proposition 2.3 and let (T, S) be any element of $M(A)$. Then $\nu(T, S) \cdot \tau x = \tau(Tx)$ and $\tau x \cdot \nu(T, S) = \tau(Sx)$ as was seen in the proof of Proposition 2.3. Therefore $\|(T, S)\| \leqslant M\|\nu(T, S)\|$ since τ is norm reducing and $\|x\| \leqslant M\|\tau x\|$ for all $x \in A$. If $M = 1$, then ν is isometric (cf. [5, Corollary 2.9]).

3. Main theorems. If B is any algebra, then Prim B will always denote the structure space of B, that is the set of all primitive ideals in B, with hull-kernel topology. Let $P \in \text{Prim } A$. Then by Theorem 2.6.6 in [8], there exists a unique element P' in Prim $D(A)$ such that $P' \cap \tau(A) = \tau(P)$. If $T \in Z(M(A))$, then $\mu T \in Z(D(A))$ from Lemma 2.2, so that $\mu T + P'$ belongs to the center of $D(A)/P'$. Notice that $D(A)/P'$ is a primitive Banach algebra and so its center reduces to the complex field. Therefore there exists a unique complex number $\Phi_T(P)$ such that $\mu T + P' = \Phi_T(P)(J + P')$.

Moreover,

$$|\Phi_T(P)| \leqslant \|\Phi_T(P)(J + P')\| = \|\mu T + P'\| \\ \leqslant \|\mu T\| \leqslant \|\mu\| \, \|T\|.$$

We thus obtain a bounded complex-valued function Φ_T on Prim A for each T in $Z(M(A))$. Let $R(A)$ be the radical of A, that is the intersection of all primitive ideals in A, and $ZM_R(A)$ the set of all $T \in Z(M(A))$ such that $T(A) \subset R(A)$. Then $ZM_R(A)$ is a closed two-sided ideal of $Z(M(A))$. Let C^b(Prim A) be the algebra of all bounded continuous complex-valued functions on Prim A.

The following result can be seen in the proof of Theorem 2.7.5 in [**8**].

LEMMA 3.1. *Let A be a Banach algebra and $Z(A)$ its center. Then for each $P \in$ Prim A with $Z(A) \not\subset P$, $P \cap Z(A) \in$ Prim $Z(A)$. If A has an identity, then $P \to P \cap Z(A)$ is a continuous map of* Prim A *into* Prim $Z(A)$.

We are now in a position to state and prove our main theorems.

THEOREM 3.2. *Let A be a complex Banach algebra with a bounded approximate identity. If the ideal center of A has a Hausdorff structure space, then the map $T \to \Phi_T$ is a continuous homomorphism of $Z(M(A))$ into C^b*(Prim A) *such that $Tx + P = \Phi_T(P)(x + P)$ for all $x \in A$ and $P \in$* Prim A, *the kernel of the homomorphism being equal to $ZM_R(A)$.*

PROOF. By the construction of Φ_T, the map $T \to \Phi_T$ is a continuous homomorphism of $Z(M(A))$ into the algebra of all bounded complex-valued functions on Prim A. We first show that $\Phi_T \in C^b$(Prim A) for each $T \in Z(M(A))$. By Theorem 2.6.6 in [**8**], the map $P \to P'$ is a homeomorphism of Prim A into Prim $D(A)$. Moreover, $M \to M \cap Z(D(A))$ is a continuous map of Prim $D(A)$ into Prim $Z(D(A))$ by Lemma 3.1. Since Prim $Z(D(A))$ is Hausdorff, $Z(D(A))$ is completely regular (cf. [**8**, Definition 2.7.1]). It follows by Theorem 3.7.1 in [**8**] that the map $Q \to \chi_Q$ is a homeomorphism of Prim $Z(D(A))$ onto Hom $Z(D(A))$. Here χ_Q denotes the nonzero homomorphism of $Z(D(A))$ onto the complex field induced by $Q \in$ Prim $Z(D(A))$ and Hom $Z(D(A))$ denotes the carrier space of $Z(D(A))$ with $Z(D(A))$-topology. We thus observe the map $P \to \chi_{P' \cap Z(D(A))}(z)$ is continuous on Prim A for each $z \in Z(D(A))$. Let $T \in Z(M(A))$ and $P \in$ Prim A. Since $\mu T + P' = \Phi_T(P)(J + P')$, we have $\mu T - \Phi_T(P)J \in P' \cap Z(D(A))$. It follows that $\Phi_T(P) = \chi_{P' \cap Z(D(A))}(\mu T)$. Then $\Phi_T \in C^b$(Prim A) by the above argument.

We next show that if $T \in Z(M(A))$, then $Tx + P = \Phi_T(P)(x + P)$ for all $x \in A$ and $P \in$ Prim A. Let $T \in Z(M(A))$, $x \in A$ and $P \in$ Prim A. Then we have

$$\tau(Tx) + P' = (\mu T)\cdot \tau x + P' = (\mu T + P')(\tau x + P')$$
$$= \Phi_T(P)(J + P')(\tau x + P') = \Phi_T(P)(\tau x + P').$$

It follows that $\tau(Tx - \Phi_T(P)x) \in P' \cap \tau(A) = \tau(P)$. Since τ is injective, we obtain that $Tx + P = \Phi_T(P)(x + P)$.

Finally we can easily see that the kernel of the map $T \to \Phi_T$ is equal to $ZM_R(A)$ from the equation $Tx + P = \Phi_T(P)(x + P)$, $x \in A$.

REMARK 3.3. Let A be as in Theorem 3.2. If the approximate identity of A is uniformly bounded by one, then the map μ is isometric [5, Corollary 2.9] and so $T \to \Phi_T$ is a norm reducing homomorphism of $Z(M(A))$ into $C^b(\text{Prim } A)$. If A is semisimple, then the map $T \to \Phi_T$ is injective. Furthermore, if A is arbitrary C^*-algebra, then the ideal center $Z(D(A))$ of A has necessarily a Hausdorff structure space because $Z(D(A))$ is also a commutative C^*-algebra.

The following result is equivalent to the Dixmier's representation theorem [6, Theorem 5].

COROLLARY 3.4. *If A is an arbitrary C^*-algebra, then the map $T \to \Phi_T$ is an isometric $*$-isomorphism of $Z(M(A))$ onto $C^b(\text{Prim } A)$.*

PROOF. Note that the map $T \to \Phi_T$ is isometric from Remark 3.3. The map is also surjective because the Dauns and Hofmann theorem [4] has showed that every function in $C^b(\text{Prim } A)$ can be realized uniquely in this way.

The following definition can be seen in the Archbold's paper [1], in case that A is a C^*-algebra.

DEFINITION 3.5. Let A be a Banach algebra and $Z(A)$ its center. Then A is said to be quasi-central if hull $Z(A) = \emptyset$. Here hull $Z(A)$ denotes the set of all primitive ideals P in A such that $Z(A) \subset P$.

THEOREM 3.6. *Let A be a quasi-central Banach algebra with a bounded approximate identity and $Z(A)$ its center. If $Z(A)$ is completely regular, then the map $T \to \Phi_T$ is a continuous homomorphism of $Z(M(A))$ into $C^b(\text{Prim } A)$.*

PROOF. As in the proof of Theorem 3.2, we only show that $\Phi_T \in C^b(\text{Prim } A)$ for each $T \in Z(M(A))$. To see this, we first show that each $P \in \text{Prim } A$ and $T \in Z(M(A))$ satisfy the following properties:

$$\chi_{P' \cap Z(D(A))}\tau | Z(A) = \chi_{P \cap Z(A)}, \tag{1}$$
$$\Phi_T(P) = \chi_{P' \cap Z(D(A))}(\mu T), \tag{2}$$
$$\Phi_T(P)\chi_{P \cap Z(A)} = \chi_{P \cap Z(A)} T | Z(A). \tag{3}$$

In fact, let $z \in Z(A)$. Since hull $Z(A) = \emptyset$, it follows that there exists $z_0 \in Z(A)$ with $z_0 \notin P$, and hence $\chi_{P \cap Z(A)}(z_0) \neq 0$. Note that $\tau(Z(A)) \subset Z(D(A))$, so that $\tau(\chi_{P' \cap Z(D(A))}(\tau z)z_0 - zz_0) \in P' \cap \tau(Z(A))$. Since $P' \cap \tau(Z(A)) = \tau(P \cap Z(A))$, we have $\chi_{P' \cap Z(D(A))}(\tau z)z_0 - zz_0 \in P \cap Z(A)$ and so $\chi_{P'\cap Z(D(A))}(\tau z) = \chi_{P\cap Z(A)}(z)$. Hence (1) holds. Notice that $\mu T - \Phi_T(P)J \in P' \cap Z(D(A))$. Then $\chi_{P'\cap Z(D(A))}(\mu T) = \Phi_T(P)\chi_{P'\cap Z(D(A))}(J) = \Phi_T(P)$ and hence (2) holds. Note that $T(Z(A)) \subset Z(A)$. Then, by (1) and (2), we have

$$\begin{aligned}\chi_{P\cap Z(A)}(Tz) &= \chi_{P'\cap Z(D(A))}(\tau(Tz))\\ &= \chi_{P'\cap Z(D(A))}(\mu T)\chi_{P'\cap Z(D(A))}(\tau z)\\ &= \Phi_T(P)\chi_{P\cap Z(A)}(z).\end{aligned}$$

Thus (3) has been shown.

By Theorem 2.7.5 in [8], $P \to P \cap Z(A)$ is a continuous map of Prim A into Prim $Z(A)$. Moreover, $I \to \chi_I$ is a homeomorphism of Prim $Z(A)$ onto Hom $Z(A)$ by Theorem 3.7.1 in [8]. Therefore, by (3), $P \to \Phi_T(P)\chi_{P\cap Z(A)}(z)$ is a continuous complex-valued function on Prim A for each $z \in Z(A)$. Suppose that Φ_T is discontinuous at some point P_0 in Prim A. Then there exists a positive number ε_0 and a net $\{P_\lambda : \lambda \in \Lambda\}$ in Prim A which converges to P_0 such that

$$|\Phi_T(P_\lambda) - \Phi_T(P_0)| \geqslant \varepsilon_0 \tag{4}$$

for all $\lambda \in \Lambda$. Since A is quasi-central, we can choose an element z_0 in $Z(A)$ such that $\chi_{P_0\cap Z(A)}(z_0) = 1$. By the above argument, the complex-valued function $P \to \chi_{P\cap Z(A)}(z_0)$ and $P \to \Phi_T(P)\chi_{P\cap Z(A)}(z_0)$ on Prim A are continuous. Then for any $\varepsilon > 0$, there exists a neighbourhood U_{P_0} of P_0 such that

$$|\chi_{P\cap Z(A)}(z_0) - 1| \leqslant \varepsilon \tag{5}$$

and

$$|\Phi_T(P)\chi_{P\cap Z(A)}(z_0) - \Phi_T(P_0)| \leqslant \varepsilon \tag{6}$$

for all $P \in U_{P_0}$. In particular, choose ε such that

$$\varepsilon = \min\left\{1/2, (4 + 4|\Phi_T(P_0)|)^{-1}\varepsilon_0\right\}.$$

Furthermore, choose $\lambda_0 \in \Lambda$ such that $P_{\lambda_0} \in U_{P_0}$ and set $\delta = \chi_{Q_0}(z_0)$, where $Q_0 = P_{\lambda_0} \cap Z(A)$. Note that $|\delta| \geqslant 1/2$ by (5). We then have

$$\begin{aligned}|\Phi_T(P_{\lambda_0}) - \Phi_T(P_0)| &\leqslant |\delta|^{-1}(|\Phi_T(P_{\lambda_0})\delta - \Phi_T(P_0)| + |\Phi_T(P_0) - \Phi_T(P_0)\delta|)\\ &\leqslant |\delta|^{-1}(\varepsilon + |\Phi_T(P_0)|\varepsilon) \quad \text{(from (5) and (6))}\\ &\leqslant \varepsilon_0/2.\end{aligned}$$

This contradicts (4). We thus obtain that Φ_T is continuous on Prim A and the proof is complete.

The author wishes to thank the referee for his helpful comments.

References

1. R. J. Archbold, *Density theorems for the center of a C^*-algebra*, J. London Math. Soc. **10** (1975), 185–197. MR **52** #1331.

2. R. C. Busby, *On structure spaces and extensions of C^*-algebras*, J. Functional Analysis **1** (1967), 370–377. MR **37** #771.

3. P. Curtis and Figa-Talamanca, *Factorization theorems for Banach algebras*, Function Algebras, (F. T. Birtel, ed.), Scott, Foresman and Co., Chicago, Ill., 1966, pp. 169–185. MR **34** #3350.

4. J. Dauns and K. H. Hofmann, *Representations of rings by continuous sections*, Mem. Amer. Math. Soc. No. 83, 1968. MR **40** #752.

5. J. W. Davenport, *Multipliers on a Banach algebra with a bounded approximate identity*, Pacific J. Math. **63** (1976), 131–135. MR **54** #931.

6. J. Dixmier, *Ideal center of a C^*-algebra*, Duke Math. J. **35** (1968), 375–382. MR **37** #5703.

7. K. Saito, *A characterization of double centralizer algebras of Banach algebras*, Sci. Rep. Niigata Univ. Ser A. **11** (1974), 5–11. MR **49** #5844.

8. C. E. Rickart, *General theory of Banach algebras*, Van Nostrand, Princeton, N.J., 1960. MR **22** #5903.

Department of Mathematics, Ibaraki University, Mito, Ibaraki 310, Japan