TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 275, Number 1, January 1983

SOME CANONICAL COHOMOLOGY CLASSES ON GROUPS OF VOLUME PRESERVING DIFFEOMORPHISMS

BY

DUSA McDUFF[1]

ABSTRACT. We discuss some canonical cohomology classes on the space $\bar{B}\mathcal{D}iff^c_{\omega 0}M$, where $\mathcal{D}iff^c_{\omega 0}M$ is the identity component of the group of compactly supported diffeomorphisms of the manifold M which preserve the volume form ω. We first look at some classes $c_k(M)$, $1 \leq k \leq n = \dim M$, which are defined for all M, and show that the top class $c_n(M) \in H^n(\bar{B}\mathcal{D}iff^c_{\omega 0}M; \mathbf{R})$ is nonzero for $M = S^n$, n odd, and is zero for $M = S^n$, n even. When $H^i_c(M; \mathbf{R}) = 0$ for $0 \leq i < n$, the classes $c_k(M)$ all vanish and a secondary class $s(M) \in H^{n-1}(\bar{B}\mathcal{D}iff^c_{\omega 0}M; \mathbf{R})$ may be defined. This is trivially zero when n is odd, and is twice the Calabi invariant for symplectic manifolds when $n = 2$. We prove that $s(\mathbf{R}^n) \neq 0$ when n is even by showing that it is one of a set of nonzero classes which were defined by Hurder in [7].

1. Statement of main results. Let M be a connected oriented n-dimensional C^∞-manifold without boundary and with smooth volume form ω. We write $\mathcal{G} = \mathcal{D}iff^c_{\omega 0}M$ for the identity component of the group of compactly supported C^∞-diffeomorphisms of M which preserve ω, with the C^∞-topology, and G for the group $\mathcal{G}$ considered with the discrete topology. The homotopy fiber of the natural map $BG \to B\mathcal{G}$ is called $\bar{B}\mathcal{G}$.

We will think of $\bar{B}\mathcal{G}$ as the realization $\mathcal{C}$ of the complex Sing $\mathcal{G}/G$. Here Sing $\mathcal{G}$ is the smooth singular complex of $\mathcal{G}$, and Sing $\mathcal{G}/G$ is its quotient by the action of G given by the multiplication on the right. Thus a p-simplex S in $\mathcal{C}$ is given by a smooth (C^∞) map $t \mapsto h_t \in \mathcal{G}$, $t \in \Delta^p$, which is well defined up to composition on the right by an element $g \in G$. Such a simplex corresponds to a codimension n foliation $\mathcal{F}(S)$ on $\Delta^p \times M$ with typical leaf $\{(t, h_t(y)) \colon t \in \Delta^p\}$. The foliation $\mathcal{F}(S)$ is transverse to the fibers $t \times M$ of the projection $p\colon \Delta^p \times M \to \Delta^p$, and is trivial, with leaves $\Delta^p \times x$, for x outside of a compact subset of M. One can also describe $\mathcal{F}(S)$ as the pull-back of the point foliation of M by the map $f\colon (t, x) \mapsto h_t^{-1}(x)$. Hence $\Omega(S) = f^*(\omega)$ is a transverse volume form for $\mathcal{F}(S)$. In other words, $\Omega(S)$ is a *closed* n-form which defines $\mathcal{F}(S)$ in the sense that a tangent vector Y of $\Delta^p \times M$ is tangent to $\mathcal{F}(S)$ if and only if $i(Y)\Omega(S) = 0$. Clearly, $\Omega(S)$ is the only transverse volume form of $\mathcal{F}(S)$ which restricts to ω on each $t \times M$.

It is easy to check that the $\mathcal{F}(S)$ fit together to give a foliation $\mathcal{F}$ of $\mathcal{C} \times M$. Similarly, the forms $\Omega(S)$ fit together to give a *canonical closed n-form* Ω on $\mathcal{C} \times M$.

Received by the editors September 21, 1981 and, in revised form, January 11, 1982.
1980 *Mathematics Subject Classification*. Primary 57T99, 57R32.
[1]Partially supported by NSF grant MCS 79 05795 A02.

0002-9947/81/0000-0101/$04.50

(By definition, a k-form Λ on $\mathcal{C} \times M$ is a collection of k-forms $\Lambda(S)$ on $\Delta^p \times M$, one for each p-simplex S in $\mathcal{C}$, such that $\Lambda(S)$ restricts to $\Lambda(S')$ on the face S' of S.)

Any form Λ on $\mathcal{C} \times M$ has a unique decomposition $\Lambda = \Lambda_0 + \Lambda_1 + \cdots + \Lambda_n$, where the form Λ_i has degree i in the M-variables. In particular,

$$\Omega = \Omega_0 + \cdots + \Omega_n$$

where Ω_n is the pull-back $\pi^*\omega$ of ω by the projection $\pi\colon \mathcal{C} \times M \to M$ and where each Ω_i, $i < n$, is compactly supported. (This means that $\Omega_i(S)$ has compact support in $\Delta^p \times M$ for all S.) Therefore Ω gives rise to *canonical cohomology classes*

$$c_k(M) \in H^k\big(\bar{B}\mathcal{D}iff^c_{\omega 0}M;\ H^{n-k}_c(M; \mathbf{R})\big)$$

for $1 \leqslant i \leqslant n$, as follows. For any k-cycle K in $\mathcal{C} = \bar{B}\mathcal{D}iff^c_{\omega 0}M$, the value of $c_k(M)$ on K is the cohomology class of the closed, compactly-supported $(n-k)$-form

$$x \mapsto \int_{K \times x} \Omega = \int_{K \times x} \Omega_{n-k}$$

on M which is obtained by integrating Ω over the fiber of the projection $\pi\colon K \times M \to M$. Explicit formulae for the $c_k(M)$ are given in Appendix 1.

Since $\pi_1\mathcal{C}$ is isomorphic to the universal cover $\widetilde{\mathcal{D}iff}^c_{\omega 0}M$ of $\mathcal{D}iff^c_{\omega 0}M$, the class c_1 corresponds to a homomorphism $\widetilde{\mathcal{D}iff}^c_{\omega 0}M \to H^{n-1}_c(M; \mathbf{R})$. Up to sign, it is just the flux homomorphism, which is defined in [**1**, II, §1] for example. See Appendix 1.

Notice that c_n vanishes when M is noncompact since $H^0_c(M; \mathbf{R}) = 0$ in this case. On the other hand $c_n(M)$ is nonzero when M is a compact Lie group. In fact, if H is a k-dimensional compact submanifold in $\mathcal{D}iff^c_{\omega 0}M$, then the "diagonal" foliation of $H \times M$ with leaves $\{(h, h(y))\colon h \in H\}$ gives rise to a k-cycle in $\mathcal{C}$. Moreover the transverse volume form on $H \times M$ is the pull-back of ω by the map $(h, y) \mapsto h^{-1}(y)$. Applying this to the action of the torus T^k on T^n one sees that $c_k(T^n) \neq 0$ for any k. Similarly taking $H \subseteq U(k)$ one has $c_n(S^n) \neq 0$ for n odd. Our first main result is

PROPOSITION 1. $c_n(S^n) = 0$ *for n even.*

This is proved in §3, together with some other calculations of $c_n(M)$.

REMARK. Fathi observed that there is a natural homomorphism

$$\psi^*\colon \tilde{H}^*(M; \mathbf{R}) \to \tilde{H}^*\big(\bar{B}\mathcal{D}iff^c_{\omega 0}M; \mathbf{R}\big)$$

which corresponds to the $c_k(M)$ by means of the formula

$$\psi^*(a)(K) = (-1)^k \langle a, c_k(M)(K)\rangle.$$

Here K is a k-cycle in $\bar{B}\mathcal{D}iff^c_{\omega 0}M$, $a \in H^k(M; \mathbf{R})$ and $\langle a, b\rangle = \int_M \alpha \wedge \beta$, where α, β are forms representing the classes a and b. It is sometimes more natural to think in terms of the homomorphism ψ^* instead of the classes c_k. Note in particular that ψ^* may be defined on the cochain level. For it is easy to check that the formula

$$\psi(\alpha)(S) = \int_{\Delta^k \times M} \pi^*\alpha \wedge \tilde{\Omega}(S),$$

where α is a k-form on M and $\tilde{\Omega}$ is the compactly supported form $\Omega - \pi^*\omega$, defines a homomorphism of the de Rham complex Λ^*M to $C^*(\mathcal{C}; \mathbf{R})$. Evidently ψ induces ψ^*.

One important fact about Ω is that $\Omega^2 = 0$. Indeed, the square of any transverse volume form is zero because locally such a form is pulled back from $\mathbf{R}^n$.

PROPOSITION 2. *If M is a noncompact manifold such that $H_c^i(M; \mathbf{R}) = 0$ for all $i < n$, then the form Ω is exact. In fact, there is a form Φ on $\mathcal{C} \times M$ such that $d\Phi = \Omega$ and $\Phi\Omega$ has compact support.*

Observe that $d(\Phi\Omega) = \Omega^2 = 0$. Hence, by integrating $\Phi\Omega$ over the fibers $t \times M$, one gets a closed $(n-1)$-form on $\mathcal{C}$ and thus an element $s(M)$ of $H^{n-1}(\bar{B}\mathcal{D}iff^c_{\omega 0}M; \mathbf{R})$. We will see in §2 that this does not depend on the choice of Φ, provided that some natural restrictions are placed on the support of Φ. Hence $s(M)$ is canonically defined. When n is odd, $\Phi\Omega = \frac{1}{2}d(\Phi^2)$ and $s(M) = 0$.

PROPOSITION 3. $s(\mathbf{R}^n) \neq 0$ *when n is even.*

COROLLARY. $s(M) \neq 0$ *when n is even.*

We will see in §3 that Propositions 1 and 3 are related. Proposition 3 is proved by showing that $s(\mathbf{R}^n)$ coincides with a nonzero class in $H^{n-1}(\bar{B}\mathcal{D}iff^c_{\omega 0}\mathbf{R}^n; \mathbf{R})$ which was discovered by Hurder [7]. His class may be described in the following way. Let $\bar{B}\Gamma^n_{sl}$ be the classifying space for codimension n foliations with transverse volume form and trivialised normal bundle. Then $\bar{B}\Gamma^n_{sl}$ is $(n-1)$-connected [4], and has $\pi_n \cong \mathbf{R}$. Let $u \in H^n(\bar{B}\Gamma^n_{sl}; \mathbf{R})$ be the "universal transverse volume form". In other word, if $\mathcal{F}$ is a foliation with transverse volume form α and trivial normal bundle which is classified by the map g, then $g^*(u) = [\alpha]$. It is shown in [10] that a map g: $S^n \to \bar{B}\Gamma^n_{sl}$ is null homotopic exactly when $g^*(u) = 0$. Therefore, if $e \in H^n(K(\mathbf{R}, n); \mathbf{R})$ is the fundamental class, there is a fibration

$$(*) \qquad \bar{\bar{B}}\Gamma^n_{sl} \xrightarrow{i} \bar{B}\Gamma^n_{sl} \xrightarrow{f} K(\mathbf{R}, n)$$

in which $\bar{\bar{B}}\Gamma^n_{sl}$ is n-connected and $f^*(e) = u$. As mentioned above, $\alpha^2 = 0$. Hence $u^2 = 0$. However, $e^2 \neq 0$ when n is even. Therefore, for even n there must be at least one nonzero element of $H^{2n-1}(\bar{\bar{B}}\Gamma^n_{sl}; \mathbf{R})$ which transgresses to e^2 in the spectral sequence of $(*)$.

One such class may be described as follows. Let g: $M \to \bar{\bar{B}}\Gamma^n_{sl}$ be a map of a manifold into $\bar{\bar{B}}\Gamma^n_{sl}$. By [4], such a map determines a foliation $\mathcal{F}$ with transverse volume form α on some bundle over M. Moreover α must be exact. Let $\alpha = d\beta$. Then the form $\beta\alpha$ is closed because α^2 is zero, and one can easily check that it represents a cohomology class $[\beta\alpha]$ in $H^{2n-1}(M; \mathbf{R})$ which depends only on the homotopy class of g. Evidently, there is a unique class $a \in H^{2n-1}(\bar{\bar{B}}\Gamma^n_{sl}; \mathbf{R})$ such that $g^*(a) = [\beta\alpha]$ for all such maps g.

LEMMA 4. *The class a transgresses to $-e^2$ in the spectral sequence of $(*)$.*

Now let

$$j\colon \left(\bar{B}\mathcal{D}iff^c_{\omega 0}\mathbf{R}^n\right) \times \mathbf{R}^n \to \bar{\bar{B}}\Gamma^n_{sl}$$

classify the canonical foliation on $(\bar{B}\mathcal{D}iff^c_{\omega 0}\mathbf{R}^n) \times \mathbf{R}^n$. (We may assume that j maps into $\bar{\bar{B}}\Gamma^n_{sl}$ since Ω is exact.) It is shown in **[8]** that the adjoint Ad j of j,

$$\text{Ad } j\colon \bar{B}\mathcal{D}iff^c_{\omega 0}\mathbf{R}^n \to \mathfrak{M}ap_{cpct}\left(\mathbf{R}^n, \bar{\bar{B}}\Gamma^n_{sl}\right) = \Omega^n\bar{\bar{B}}\Gamma^n_{sl},$$

induces an isomorphism on integer homology when $\mathbf{R}^n$ has infinite ω-volume. (If $\mathbf{R}^n$ has finite ω-volume one can either use **[9]** or, preferably argue as in §2 below.)

Let h be the composite

$$H^{2n-1}\left(\bar{\bar{B}}\Gamma^n_{sl}\right) \to H^{n-1}\left(\Omega^n\bar{\bar{B}}\Gamma^n_{sl}\right) \overset{(\text{Ad } j)^*}{\to} H^{n-1}\left(\bar{B}\mathcal{D}iff^c_{\omega 0}\mathbf{R}^n\right).$$

Hurder pointed out that the class a is spherically supported. This implies that its image in $H^{n-1}(\Omega^n\bar{\bar{B}}\Gamma^n_{sl})$ is nonzero. Hence $h(a)$ is nonzero. We will see in §2 that $h(a) = s(\mathbf{R}^n)$. It follows that $s(\mathbf{R}^n)$ is also nonzero.

When $n = 2$ the class $s(\mathbf{R}^2)$ is a multiple of the Calabi invariant. This invariant is an element of $H^1(\bar{B}\mathcal{D}iff^c_{\sigma 0}M; \mathbf{R})$, where $\mathcal{D}iff^c_{\sigma 0}M$ is the identity component of the group of compactly supported symplectic diffeomorphisms of M, and is defined for certain noncompact M. See **[1**, II, §4 and **11]**. The symplectic case is considered further in Appendix 3. In particular we show that the restriction of $s(\mathbf{R}^{2m})$ to $\bar{B}\mathcal{D}iff^c_{\sigma 0}\mathbf{R}^{2m}$ is zero when $m > 1$.

All the classes which are mentioned in this paper are smooth and so may be defined on the Lie algebra level. This is discussed in Appendix 2.

It is a pleasure to acknowledge helpful conversations with A. Fathi, D. Gottlieb, A. Haefliger and especially with S. Hurder. Also I wish to thank I.H.E.S. for its kind hospitality during the writing of this paper.

2. The class s. Let A^* be the algebra of all smooth forms on $\mathcal{C} \times M$, and let A^*_c be the subalgebra of forms which are compactly supported with respect to M. We will write $A^{r,s}$, resp. $A^{r,s}_c$, for the subspace of $A^{r+s} = A^k$, resp. A^k_c, consisting of all k-forms with degree r in the T-variables. We will identify Δ^p with $\{(t_1,\ldots,t_p) \in \mathbf{R}^p\colon 0 \leq t_1 \leq \cdots \leq t_p \leq 1\}$. Then the restriction of $\Lambda \in A^{r,s}_c$ to the p-simplex S in $\mathcal{C}$ is a form

$$(\sharp) \qquad \Lambda(S) = \sum dt_{i_1} \wedge \cdots \wedge dt_{i_r} \wedge \alpha_{i_1\cdots i_r}(t)$$

on $\Delta^p \times M$, where each $\alpha_{i_1\cdots i_r}(t)$, $t \in \Delta^p$, is a smooth family of compactly supported s-forms on M. Note also that the differential d on A^* is the sum $d = d_T + d_M$, where d_M and d_T differentiate with respect to the variables in M and Δ^p respectively.

LEMMA 5. *If* $H^s_c(M; \mathbf{R}) = 0$, *the sequence*

$$A^{r,s-1}_c \overset{d_M}{\to} A^{r,s}_c \overset{d_M}{\to} A^{r,s+1}_c$$

is exact.

PROOF. This is routine. Suppose that $\Lambda \in A^{r,s}_c$ is in $\ker d_M$, and suppose inductively that a solution of the equation $d_M\Psi = \Lambda$ has been constructed on $\mathcal{C}_{p-1} \times M$, where $\mathcal{C}_{p-1}$ is the $(p-1)$-skeleton of $\mathcal{C}$. Then we must define $\Psi(S)$ for each p-simplex S. The conditions on Λ and M imply that the forms $\alpha_{i_1\cdots i_r}(t)$ in the expression (♯) for Λ are exact for each t. The problem now is to find smooth families $\beta_{i_1\cdots i_r}(t)$ of compactly-supported $(s-1)$-forms on M such that $d_M\beta_{i_1\cdots i_r}(t)=\alpha_{i_1\cdots i_r}(t)$. Also, setting

$$\Psi(S)=\sum dt_{i_1}\wedge\cdots\wedge dt_{i_r}\wedge\beta_{i_1\cdots i_r}(t)$$

we need $\Psi(S)$ to agree with the previously defined Ψ on $\partial\Delta^p\times M$. However one can always find suitable families $\beta(t)$ by using de Rham's theorem with parameters. □

PROPOSITION 6. *If $H_c^q(M;\mathbf{R})=0$ for $0<q\leqslant k$, then any closed form $\Lambda\in A_c^p$ which has no component in $A_c^{p-s,s}$ for $s>k$ may be written*

$$\Lambda=d\Psi+\chi_0,$$

where $\Psi\in A_c^{p-1}$ and $\chi_0\in A_c^{p,0}$.

PROOF. Since $H_c^n(M;\mathbf{R})=\mathbf{R}$, we must have $k<n$. Then $\Lambda=\Lambda_0+\cdots+\Lambda_k$, where $\Lambda_i\in A_c^{p-i,i}$. Because Λ is closed we have

$$d_M\Lambda_k=0,\quad d_T\Lambda_0=0\quad\text{and}\quad d_M\Lambda_{q-1}+d_T\Lambda_q=0,\qquad 0<q\leqslant k.$$

The argument is now completed by a standard diagram chase in the double complex:

$$\begin{array}{ccccccc}
A_c^{0,2} & \xrightarrow{d_T} & A_c^{1,2} & \rightarrow & \cdots & & \\
d_M\uparrow & & d_M\uparrow & & \uparrow & & \\
A_c^{0,1} & \xrightarrow{d_T} & A_c^{1,1} & \xrightarrow{d_T} & A_c^{2,1} & \rightarrow & \cdots \\
d_M\uparrow & & d_M\uparrow & & d_M\uparrow & & \\
A_c^{0,0} & \xrightarrow{d_T} & A_c^{1,0} & \xrightarrow{d_T} & A_c^{2,0} & \rightarrow & \cdots
\end{array}$$ □

COROLLARY 7. *If in addition M is noncompact, then $\chi_0=0$ and $\Lambda=d\Psi$.*

PROOF. Observe that χ_0 is closed. Hence,

$$\chi_0(S)=\sum\gamma_{i_1\cdots i_p}(t,x)\,dt_{i_1}\wedge\cdots\wedge dt_{i_p},$$

where the functions $\gamma_{i_1\cdots i_p}(t,x)$ are constant with respect to $x\in M$. Since they are also compactly supported, they must vanish when M is noncompact. □

PROOF OF PROPOSITION 2. We must show that the canonical form Ω is exact when $H_c^q(M;\mathbf{R})=0$ for $0\leqslant q<n$. But $\Omega=\tilde{\Omega}+\Omega_n$ where $\Omega_n=\pi^*(\omega)$ is exact, and where $\tilde{\Omega}=\Omega_0+\cdots+\Omega_{n-1}$ is closed. Hence Ω is exact by Corollary 7. □

Thus the equation $d\Phi=\Omega$ has a solution $\Phi=\tilde{\Phi}+\Phi_n$ where $\tilde{\Phi}\in A_c^{n-1}$ satisfies $d\tilde{\Phi}=\tilde{\Omega}$ and where $d\Phi_n=d_M\Phi_n=\Omega_n$. Clearly $\Phi\Omega\in A_c^{2n-1}$. Also $d(\Phi\Omega)=\Omega^2=0$. Therefore, we may define the cohomology class $s(M)$ as a cochain by setting

$$s(M)(S)=\int_{\Delta^{n-1}\times M}\Phi\Omega$$

for each $(n-1)$-simplex S. Since $\Phi\Omega=\frac{1}{2}d(\Phi^2)$ when n is odd, the class $s(M)$ is zero in this case. (Observe that Φ^2 is compactly supported.) When n is even, it will sometimes be useful to define s using the form $\tilde{\Phi}(\Omega+\Omega_n)=\Phi\Omega-d(\tilde{\Phi}\Phi_n)$. This alternative definition makes it clear that the cohomology class s does not depend on the choice of $\tilde{\Phi} \in A_c^{n-1}$ or of Φ_n, provided that $d\tilde{\Phi} = \tilde{\Omega}$ and $d\Phi_n = \Omega_n$.

In the remainder of this section, we will fill in the details of the proof that $s(\mathbf{R}^n) \neq 0$ for n even which was sketched in §1. First we will prove Lemma 4 which says that the class $a \in H^{2n-1}(\bar{\bar{B}}\Gamma_{sl}^n)$ transgresses to $-e^2$ in the spectral sequence of the fibration

$$\bar{\bar{B}}\Gamma_{sl}^n \xrightarrow{i} \bar{B}\Gamma_{sl}^n \xrightarrow{f} K(\mathbf{R}, n).$$

PROOF OF LEMMA 4. Recall that the transgression τ is the composite $(\bar{f}^*)^{-1} \circ \delta$ where

$$H^{2n-1}\left(\bar{\bar{B}}\Gamma_{sl}^n\right) \xrightarrow{\delta} H^{2n}\left(\bar{B}\Gamma_{sl}^n, \bar{\bar{B}}\Gamma_{sl}^n\right) \xleftarrow{\bar{f}^*} H^{2n}(K(\mathbf{R}, n), *).$$

Therefore, because $e^2 \in \operatorname{Im} \tau$ it will suffice to show that $\delta(a) = -\bar{f}^*(e^2)$. This will follow if we show that $\delta(g^*a) = -g^*\bar{f}^*(e^2)$ for any map g of a compact manifold with boundary $(M, \partial M)$ into the pair $(\bar{B}\Gamma_{sl}^n, \bar{\bar{B}}\Gamma_{sl}^n)$.

A map g: $M \to \bar{B}\Gamma_{sl}^n$ corresponds to a foliation $\mathcal{F}$ of some bundle E over M with transverse volume form $\tilde{\alpha}$. We will identify M with the zero section of E and will put $\alpha = \tilde{\alpha} \mid M$. Then the restriction of α to ∂M is exact because $g(\partial M) \subseteq \bar{\bar{B}}\Gamma_{sl}^n$. Let β be a form on M such that $d\beta = \alpha$ on ∂M. Then g^*a is by definition the class $[\beta\alpha]$ on ∂M. Since $\alpha = d\beta$ on ∂M and $\alpha^2 = 0$, we have

$$\delta(g^*a) = \delta[\beta\alpha] = \delta[2\beta\alpha - \beta d\beta] = \left[-(\alpha - d\beta)^2\right]$$

in $H^{2n}(M, \partial M)$. It follows that the class $[(\alpha - d\beta)^2]$ is independent of the choice of β. On the other hand, it is easy to check that as β varies the class $[\alpha - d\beta]$ runs over the set of elements of $H^n(M, \partial M)$ which map to $[\alpha] = g^*f^*(e)$ in $H^n(M)$. Therefore $g^*\bar{f}^*(e) = [\alpha - d\beta]$ for some β, and $g^*\bar{f}^*(e^2) = -\delta(g^*a)$ as required. □

The next step is to show that a is spherically supported. In fact, let λ be a nonzero element of $\pi_n(\bar{B}\Gamma_{sl}^n)$ and let $\mu \in \pi_{2n-1}(\bar{\bar{B}}\Gamma_{sl}^n)$ be taken by i_* to the Whitehead product $[\lambda, \lambda] \in \pi_{2n-1}(\bar{B}\Gamma_{sl}^n)$. Then $\langle u, \mathcal{H}(\lambda)\rangle \neq 0$, where $\mathcal{H}$ is the Hurewicz homomorphism and $u \in H^n(\bar{B}\Gamma_{sl}^n; \mathbf{R})$ is the universal transverse volume form. It is now easy to check that $\langle a, \mathcal{H}(\mu)\rangle \neq 0$: for example, see **[6]**.

Finally we must show that $h(a) = s(\mathbf{R}^n)$. Consider the commutative diagram

$$\begin{array}{ccc} H^{2n-1}\left(\bar{\bar{B}}\Gamma_{sl}^n\right) & \to & H^{n-1}\left(\Omega^n\bar{\bar{B}}\Gamma_{sl}^n\right) \\ \downarrow j^* & & \downarrow (Ad\, j)^* \\ H^{2n-1}(\mathcal{C} \times \mathbf{R}^n) & \xrightarrow{\mathcal{I}} & H^{n-1}(\mathcal{C}) \end{array}$$

where $\mathcal{I}$ is integration over the fiber $t \times \mathbf{R}^n$ and where j is as in §1. It is clear from the definition of a that $j^*(a)$ is represented by the form $\Phi\Omega$ on $\mathcal{C} \times \mathbf{R}^n$. Hence $h(a) = s(\mathbf{R}^n)$ as claimed.

This completes the proof of Proposition 3. The results of **[9]** were used here in order to show that j_* is an isomorphism when $\mathbf{R}^n$ has finite ω-volume. This may be avoided by arguing as follows. Let $\mathcal{G}_\lambda$ be the subgroup of $\mathcal{D}iff_{\omega 0}^c\mathbf{R}^n$ consisting of diffeomorphisms with support in the open ball of radius λ. If $\mathrm{vol}_{\tilde\omega}\mathbf{R}^n \leq \mathrm{vol}_{\omega}\mathbf{R}^n = \infty$, there is by [2] an embedding i: $\mathbf{R}^n \hookrightarrow \mathbf{R}^n$ such that $i^*\omega = \tilde\omega$. We may also assume that $\mathcal{G}_\lambda \subset i_*(\mathfrak{D}iff^c_{\tilde\omega 0}\mathbf{R}^n)$ for some $\lambda > 0$. Therefore, in order to show that $s(\mathbf{R}^n, \tilde\omega) \neq 0$ it suffices to show that the restriction s_λ of $s(\mathbf{R}^n, \omega)$ to $\overline{B}\mathcal{G}_\lambda$ is nonzero for all λ. Because $\mathfrak{D}iff^c_{\omega 0}\mathbf{R}^n = \varinjlim \mathcal{G}_\lambda$, some s_λ is certainly nonzero. And it is easy to check that the isomorphism r: $\mathcal{G}_\mu \to \mathcal{G}_\lambda$ which is induced by multiplication by $\mu^{-1}\lambda$ takes s_μ to a nonzero multiple of s_λ. A similar argument proves the corollary to Proposition 3.

3. The class c_n. We first prove Proposition 1 which states that $c_n(S^n) = 0$ when n is even.

PROOF OF PROPOSITION 1. By Proposition 6 there are forms $\tilde\Phi$ and χ_0 on $\mathcal{C} \times S^n$ such that $\tilde\Omega = d\tilde\Phi + \chi_0$ and $\chi_0 \in A^{n,0}_c$. Note also that if K is any n-cycle in $\mathcal{C}$, the integral of Ω over $K \times x$ is independent of x. Hence

$$c_n(K) = \int_{K\times x} \Omega = \lambda \int_{K\times S^n} \Omega\Omega_n$$

where $\lambda^{-1} = \int_{S^n}\omega$. But

$$\Omega\Omega_n = (\tilde\Omega - \chi_0 + \Omega_n)\Omega_n + \chi_0\Omega_n = d(\tilde\Phi\Omega_n) + \chi_0\Omega_n.$$

Also, because $\tilde\Omega(\Omega + \Omega_n) = \Omega^2 - \Omega_n^2 = 0$ when n is even, we have

$$2\chi_0\Omega_n = (\chi_0 - \tilde\Omega)(\Omega + \Omega_n) - \chi_0\tilde\Omega = -d(\tilde\Phi(\Omega + \Omega_n)) - \chi_0\tilde\Omega.$$

Therefore

$$2c_n(K) = -\lambda \int_{K\times S^n} \chi_0\tilde\Omega = 0,$$

since all the terms in $\chi_0\tilde\Omega$ have degree $< n$ in the x-variables. □

It would be interesting to understand which compact manifolds M have $c_n(M) = 0$. Clearly the above proof applies when $H^i(M; \mathbf{R}) = 0, 0 < i < n$, and n is even. The examples given in §1 of manifolds such that $c_n(M) \neq 0$ were constructed using the action of $\mathfrak{D}iff_{\omega 0}M$ on M and were really examples with $i^*c_n(M) \neq 0$ where i: $\mathfrak{D}iff_{\omega 0}M \to \overline{B}\mathfrak{D}iff_{\omega 0}M$ is the natural map. In fact, a cycle γ: $K \to \mathfrak{D}iff_{\omega 0}M$ in $\mathfrak{D}iff_{\omega 0}M$ gives rise to a foliation on $K \times M$ with leaves $\{(k, \gamma(k)y) : k \in K\}$. Its transverse volume form is the pull-back of ω by the map $(k, y) \mapsto \gamma(k)^{-1}y$. It follows that the value of the class $i^*c_n(M)$ on the cycle K is the constant function on M which equals $\langle e^*[\omega], K\rangle$, where e: $K \to M$ is the evaluation map $k \mapsto \gamma(k)^{-1}y$. Gottlieb proves in [3, Theorem A] that $\chi(M) \cdot e^*([\omega]) = 0$, where $\chi(M)$ is the Euler characteristic of M. Hence we have shown

LEMMA 8. *If M is a compact manifold with $\chi(M) \neq 0$, then $i^*c_n(M) = 0$.*

In particular, this implies that $i^*c_2(X) = 0$ when X is an oriented surface of genus > 1.

LEMMA 9. *If X is an oriented surface of genus > 0, then $c_2(X) \neq 0$.*

PROOF. Let $Y = X - \{x_0\}$. We will write $\mathcal{C}_X$ for $\overline{B}\mathfrak{D}iff_{\omega 0}X$ and $\mathcal{C}_Y$ for $\overline{B}\mathfrak{D}iff^c_{\tilde\omega 0}Y$ where $\tilde\omega = \omega \mid Y$. Let p: $\mathcal{C}_X \to \overline{B}\Gamma^2_{sl}$ classify the germ at $\mathcal{C}_X \times x_0$ of the canonical foliation of $\mathcal{C}_X \times X$. It is shown in [9] that the homotopy fiber of p has the same integral homology as $\mathcal{C}_Y$. Therefore there is a spectral sequence

$$H^r(\bar{B}\Gamma^2_{sl}; H^s(\mathcal{C}_Y)) \Rightarrow H^{r+s}(\mathcal{C}_X)$$

where coefficients in $\mathbf{R}$ are understood. Since $\bar{B}\Gamma^2_{sl}$ is 1-connected, this gives rise to an exact sequence

$$(\sharp) \qquad 0 \to H^1(\mathcal{C}_X) \xrightarrow{j^*} H^1(\mathcal{C}_Y) \xrightarrow{\tau} H^2(\bar{B}\Gamma^2_{sl}) \xrightarrow{p^*} H^2(\mathcal{C}_X)$$

where τ is the transgression and j^* is induced by the inclusion $Y \subseteq X$.

Banyaga shows in [1] that $H_1(\mathcal{C}_X; \mathbf{Z}) \cong H^1(X; \mathbf{R})$. Further, because the form $\langle \alpha, \beta \rangle = \int_Y \alpha \wedge \beta$ does not vanish identically for $\alpha, \beta \in H^1_c(Y; \mathbf{R})$, the results of Rousseau [11] imply that $H_1(\mathcal{C}_Y; \mathbf{Z}) \cong H^1_c(Y; \mathbf{R})$. Thus there is a commutative diagram

$$\begin{array}{ccc} H_1(\mathcal{C}_Y; \mathbf{Z}) & \to & H^1_c(Y; \mathbf{R}) \\ \downarrow & & \downarrow \\ H_1(\mathcal{C}_X; \mathbf{Z}) & \to & H^1(X; \mathbf{R}) \end{array}$$

where the horizontal maps are isomorphisms given by the flux and the vertical maps are induced by the inclusion of Y in X. It follows that the map j^* in the exact sequence ($\sharp$) is an isomorphism and that p^* is injective. But if $u \in H^2(\bar{B}\Gamma^2_{sl})$ is the "universal transverse volume form", p^*u is represented by the restriction of Ω to $\mathcal{C}_X \times x_0$ and so equals $c_2(X)$. Hence $c_2(X) \neq 0$ as claimed. □

In general, if $Y = X - x_0$ one has an exact sequence

$$H^{n-1}(\mathcal{C}_Y) \xrightarrow{\tau} H^n(\bar{B}\Gamma^n_{sl}) \xrightarrow{p^*} H^n(\mathcal{C}_X)$$

in which $p^*u = c_n(X)$. Therefore the vanishing of $c_n(X)$ implies that u is in the image of τ. In particular, consider the case $X = S^n$, for n even.

LEMMA 10. *$\tau(s(\mathbf{R}^n))$ is a nonzero multiple of u.*

PROOF. Recall that τ is the composite $(\bar{p}^*)^{-1} \circ \delta$ where

$$H^{n-1}(\mathcal{C}_Y) \xrightarrow{\delta} H^n(\mathcal{C}_X, \mathcal{C}_Y) \xleftarrow{\bar{p}^*} H^n(\bar{B}\Gamma^n_{sl}).$$

Therefore, it will suffice to show that $\delta(s(\mathbf{R}^n))$ is a nonzero multiple of $\bar{p}^*u$. Let $\tilde{\Phi}$ and χ_0 be forms on $\mathcal{C}_X \times X$ as in the proof of Proposition 1. Clearly we may assume that the restriction $\tilde{\Phi}_Y$ of $\tilde{\Phi}$ to $\mathcal{C}_Y \times Y$ is compactly supported and satisfies $d\tilde{\Phi}_Y = \tilde{\Omega}$. Thus $\tilde{\Phi}$ is zero on some neighbourhood of $\mathcal{C}_Y \times x_0$ in $\mathcal{C}_Y \times X$, and $\chi_0 = 0$ on $\mathcal{C}_Y \times X$. The class $\bar{p}^*u$ is represented by the restriction of the form Ω to $(\mathcal{C}_X \times x_0, \mathcal{C}_Y \times x_0)$. Therefore, if L is an n-cycle in $(\mathcal{C}_X, \mathcal{C}_Y)$ we have

$$\bar{p}^*u(L) = \int_{L \times x_0} \Omega = \int_{L \times x_0} (d\tilde{\Phi} + \chi_0) + \Omega_n = \int_{\partial L \times x_0} \tilde{\Phi} + \int_{L \times x_0} (\chi_0 + \Omega_n).$$

But $\tilde{\Phi} = 0$ on $\partial L \times x_0$ since $\partial L \subset \mathcal{C}_Y$. Also $\int_{L\times x_0}\Omega_n = 0$. Therefore, because χ_0 is independent of x (see Corollary 7) we have

$$\tilde{p}^*u(L) = \int_{L\times x_0}\chi_0 = \lambda\int_{L\times X}\chi_0\Omega_n \quad \text{where } \lambda^{-1} = \int_X \omega.$$

But

$$\delta(s(\mathbf{R}^n))(L) = s(\mathbf{R}^n)(\partial L) = \int_{\partial L\times Y}\tilde{\Phi}(\Omega + \Omega_n) = \int_{\partial L\times X}\tilde{\Phi}(\Omega + \Omega_n)$$

$$= \int_{L\times X}(\tilde{\Omega} - \chi_0)(\Omega + \Omega_n) = -\int_{L\times X}2\chi_0\Omega_n$$

as in the calculation of $c_n(X)$ in Proposition 1. □

This result suggests that one should be able *to extend s to a natural nonzero class $s(Y) \in H^{n-1}(\bar{B}\mathfrak{D}iff^c_{\omega 0}Y; \mathbf{R})$ which is defined for all even dimensional manifolds of the form $Y = X - x_0$, where X is a compact manifold with $c_n(X) = 0$.* (To say that s is natural means of course that $j^*s(Y') = s(Y)$ for all embeddings j: $(Y, \omega) \hookrightarrow (Y', \omega')$.) Similarly, one might expect that s would always be zero on odd dimensional manifolds and hence that $c_n(X) \neq 0$ *for all odd dimensional X*. Note that these suggestions are consistent with our results in dimension 2, for Rousseau's work says exactly that the only noncompact 2-dimensional manifold on which s can be defined is $\mathbf{R}^2$.

Appendix 1. Explicit formulae for the c_k. Consider a p-simplex S in $\mathcal{C}$. Let $\tilde{X}_1(t),\ldots,\tilde{X}_p(t)$ be the vector fields on $\Delta^p \times M$ which are tangent to the leaves of the foliation $\mathfrak{F}(S)$ and which lie above the fields $\partial/\partial t_1,\ldots,\partial/\partial t_p$ on $\Delta^p = \{(t_1,\ldots,t_p)$: $0 \leq t_1 \leq \cdots \leq t_p \leq 1\}$. Then for each $t \in \Delta^p$ we may write

$$\tilde{X}_i(t) = X_i(t) + \partial/\partial t_i,$$

where $X_i(t)$ is an ω-preserving vector field on M. (Compare [**1**, §I] where it is shown that $X_i(t, x) = \partial h_t/\partial t_i(h_t^{-1}(x))$.) The vectors $[\tilde{X}_i(t), \tilde{X}_j(t)]$ are tangent to the leaves of $\mathfrak{F}(S)$ and project to $[\partial/\partial t_i, \partial/\partial t_j] = 0$ on Δ^p. Since the projection of $\Delta^p \times M$ on Δ^p maps each leaf of $\mathfrak{F}(S)$ diffeomorphically onto Δ^p, it follows that $[\tilde{X}_i(t), \tilde{X}_j(t)] = 0$. This proves

LEMMA A1 [**1**, Proposition I.1.1]. *For each $t \in \Delta^p$ we have*

$$[X_i(t), X_j(t)]_M = \frac{\partial}{\partial t_j}(X_i(t)) - \frac{\partial}{\partial t_i}(X_j(t)).$$

For short we will write this equation as

$$[X_i, X_j] = \partial_j X_i - \partial_i X_j.$$

Next, suppose that $Y_1,\ldots,Y_k$ are ω-preserving vector fields on M and consider the $(n-k)$-form

$$\omega(Y_1,\ldots,Y_k;\ldots) = i(Y_k)\cdots i(Y_1)\omega$$

on M. When $k = 1$ this form is closed. In general one has

LEMMA A2.

$$d_M\omega(Y_1,\ldots,Y_k;\ldots) = \sum_{i<j}(-1)^{i+j+k}\omega\big([Y_i, Y_j], Y_1,\ldots,\hat{i},\hat{j},\ldots,Y_k;\ldots\big).$$

PROOF. This follows easily by induction on k, using the formulae

$$di(X) + i(X)d = \mathfrak{L}_X,$$

and

$$\mathfrak{L}_X(\omega(Y_1,\ldots,Y_k;\ldots)) = \omega([X, Y_1], Y_2,\ldots,Y_k;\ldots) + \cdots + \omega(Y_1,\ldots,[X, Y_k];\ldots). \quad \square$$

PROPOSITION A3. *For each p-simplex S we have*

$$\Omega_{n-k}(S) = (-1)^k \sum_{1\leqslant i_1<\cdots<i_k\leqslant p} dt_{i_1}\wedge\cdots\wedge dt_{i_k}\wedge\omega(X_{i_1},\ldots,X_{i_k};\ldots)$$

where the fields $X_i(t)$ are as defined above.

PROOF. Let $\Lambda(S) = \Lambda_0(S) + \cdots + \Lambda_n(S)$ be the form defined by the right-hand side of the above equations. Note that $\Lambda_k(S) = 0$ if $k < n - p$. We must check that $\Lambda(S)$ is closed, that it defines $\mathfrak{F}(S)$ and that it restricts to ω on each fiber $t \times M$. The last two statements are easy to verify, and will be left to the reader. Since $\Lambda_n(S) = \pi^*(\omega)(S)$ is closed, the first statement is equivalent to the equations

$$d_T\Lambda_k(S) + d_M\Lambda_{k-1}(S) = 0 \quad \text{for } 0 \leqslant k \leqslant n.$$

These follow easily from Lemmas A1 and A2. For example,

$$\begin{aligned} d_T\Lambda_{n-1}(S) &= d_T\Big(-\sum_i dt_i\wedge\omega(X_i;\ldots)\Big) \\ &= -\sum_{i\neq j} dt_j\wedge dt_i\wedge\omega(\partial_j X_i;\ldots) \\ &= \sum_{i<j} dt_i\wedge dt_j\wedge\omega\big([X_i, X_j];\ldots\big) = -d_M\Lambda_{n-2}(S). \quad \square \end{aligned}$$

COROLLARY A4. *For each k-cycle K in $\mathcal{C}$ we have*

$$c_k(M)(K) = \int_K \Omega_{n-k} = (-1)^k \sum_{S\in K}\int_{\Delta^k} dt_1\wedge\cdots\wedge dt_k\wedge\omega(X_1,\ldots,X_k;\ldots).$$

It follows immediately that $-c_1(M)$ is the flux homomorphism.

Appendix 2. Lie algebra cohomology. In [5] Haefliger describes spaces such as $\overline{B}\mathfrak{D}iff^c_{\omega 0}M$ in a slightly different context, emphasizing their connection with Lie algebras. In this appendix we will translate our results into his language. I am grateful to Haefliger for explaining how this may be done.

Let $\mathfrak{g}_M$ be the Lie algebra of compactly supported vector fields on M which preserve ω. Consider the double complex $C^{r,s}(M) = C^r(\mathfrak{g}_M; \Lambda^s(M))$ of r-cochains on $\mathfrak{g}_M$ with values in the s-forms on M where $\mathfrak{g}_M$ acts trivially on Λ^*M, and let

$C^*(M)$ be the associated total complex. As in [5], one can construct a universal characteristic homomorphism

$$\chi\colon C^{r,s}(M) \to A_d^{r,s} \subseteq A^{r,s}$$

where $A^{r,s}$ is as in §2 and $A_d^{r,s}$ is its subcomplex of forms smooth under deformation. Define $\bar{\Omega}_{n-k} \in C^{k,n-k}$ by

$$\bar{\Omega}_{n-k}(X_1,\ldots,X_k) = (-1)^k \omega(X_1,\ldots,X_k;\ldots)$$

and let $\bar{\Omega} = \Sigma\,\bar{\Omega}_{n-k} \in C^n(M)$. Then $\bar{\Omega}$ is closed. Moreover, because $\theta(\partial/\partial t_i) = X_i(t)$, the calculations of Appendix 1 show that $\chi(\bar{\Omega}) = \Omega$. Thus $\bar{\Omega}$ is the Lie algebra analogue of Ω.

Since $\bar{\Omega}$ is exact when $H_c^i(M;\mathbf{R}) = 0$, $i < n$, one can easily construct a class $\bar{s} \in H^{n-1}(\mathfrak{g}_M;\mathbf{R})$ which is taken by χ to s. There are also classes $\bar{c}_k \in H^k(\mathfrak{g}_M;\, H_c^{n-k}(M))$ which correspond to the c_k. As before, they come from a homomorphism

$$\bar{\psi}^*\colon H^*(M) \to H^*(\mathfrak{g}_M;\mathbf{R})$$

which is induced by the cochain map

$$\bar{\psi}\colon \Lambda^* M \to C^*(\mathfrak{g}_M;\mathbf{R})$$

given by

$$\bar{\psi}\,(\beta)(X_1,\ldots,X_k) = \int_M \beta \wedge \omega(X_1,\ldots,X_k;\ldots), \quad \text{for } k \geq 1.$$

Appendix 3. The symplectic case. Let σ be a symplectic form on M and set $\omega = \sigma^m$ where $\dim M = 2m = n$. Then $\mathcal{C}_\sigma = \bar{B}\mathfrak{D}iff^c_{\sigma 0}M$ is a subcomplex of $\mathcal{C}$. The arguments of §1 show that $\mathcal{C}_\sigma \times M$ carries a canonical symplectic form Σ which is induced locally from $\mathbf{R}^{2m}$. Clearly $\Sigma^m = \Omega$ and $\Sigma^{m+1} = 0$. Moreover, one can show as in Appendix 1 that $\Sigma = \Sigma_0 + \Sigma_1 + \Sigma_2$ where $\Sigma_2 = \pi^*(\sigma)$, and

$$\Sigma_1(S) = -\sum_{1 \leq i \leq p} dt_i \wedge \sigma(X_i;\,\cdot),$$

$$\Sigma_0(S) = \sum_{1 \leq i < j \leq p} \sigma(X_i,\, X_j)\, dt_i \wedge dt_j$$

on any p-simplex S in $\mathcal{C}_\sigma$. The form Σ defines classes similar to the c_k.

If M is noncompact and if $H_c^1(M;\mathbf{R}) = 0$, then Corollary 7 implies that there is a form $\tilde{\Psi}$ in $A_c^{1,0}$ such that $d\tilde{\Psi} = \Sigma_0 + \Sigma_1$. It is easy to check that on any 1-simplex S we have $\tilde{\Psi}(S) = \alpha(t, x)\, dt$, where, for each $t \in I$ the function $\alpha(t, x)$ is the unique compactly supported function on M such that $d_M\alpha(t, x) = \sigma(X(t);\,\cdot)$. The Calabi class $R \in H^1(\bar{B}\mathfrak{D}iff^c_{\sigma 0}M;\mathbf{R})$ is then given by

$$R(S) = \int_{I \times M} \alpha(t, x)\, dt \wedge \sigma^m = \frac{1}{2}\int_{I \times M} \tilde{\Psi}(\Omega + \Omega_n)$$

for each 1-simplex S. See [1, II, §4]. This should be compared with the formula $s(M)(S) = \int_{\Delta^{n-1} \times M} \tilde{\Psi}(\Omega + \Omega_n)$ for $s(M)$. Note that these formulae agree up to a constant when $m = 1$. Observe also that the form $\tilde{\Psi}(\Omega + \Omega_n)$ is not closed. It defines a cohomology class as above because the part of $d(\tilde{\Psi}(\Omega + \Omega_n))$ which lies in $A_c^{2,n}$ is d_M-exact and so has zero integral over M. To see this one uses the formula $2\Sigma_0\Sigma_2^m + m\Sigma_1^2\Sigma_2^{m-1} = 0$, which is the part of the equation $\Sigma^{m+1} = 0$ concerning forms in $A^{2,n}$, and observes that $\Sigma_1 = d_M\tilde{\Psi}$.

Let $\Lambda_j = \Sigma^j + \Sigma^{j-1}\Sigma_2 + \cdots + \Sigma_2^j$. Then $\tilde{\Psi}\Lambda_m$ is closed and its component in $A_c^{1,n}$ equals $(m+1)\tilde{\Psi}\Sigma_2^m$. Hence the class $(m+1)R$ may be defined by integrating $\tilde{\Psi}\Lambda_m$ over M. More generally one gets classes $R_j \in H^{2j+1}(\bar{B}\mathcal{D}iff_{\sigma 0}^c M; \mathbf{R})$ by integrating $\tilde{\Psi}\Lambda_m\Sigma^j$ over $M, 0 \leqslant j \leqslant m$. Notice also that $d(\tilde{\Psi}\Lambda_{m-1}) = \Sigma^m - \Sigma_2^m$. Hence $\Omega = \Sigma^m$ is exact on $\mathcal{C}_\sigma \times M$ and one can define a class s_σ in $H^{2m-1}(\bar{B}\mathcal{D}iff_{\sigma 0}^c M; \mathbf{R})$ which corresponds to s. It is not hard to check that $s_\sigma = R_{m-1}$. Since the classes R_j, $j \geqslant 1$, vanish when Σ_2 and hence Σ are exact, this shows that the *restriction of* $s(\mathbf{R}^{2m})$ *to* $\bar{B}\mathcal{D}iff_{\sigma 0}^c\mathbf{R}^{2m}$ *is zero if* $m > 1$.

Note finally that the R_j may be defined on $\bar{B}\mathcal{D}iff_{\sigma 0}^{\Phi c}M$ for any symplectic, noncompact M, where $\mathcal{D}iff_{\sigma 0}^{\Phi c}M$ is the kernel of the flux homomorphism $\mathcal{D}iff_{\sigma 0}^c M \to H_c^1(M; \mathbf{R})/\Gamma$.

References

1. A. Banyaga, *Sur la structure du groupe des difféomorphismes qui préservent une forme symplectique*, Comment. Math. Helv. **53** (1978), 174–227.

2. R. E. Greene and K. Shiohoma, *Diffeomorphisms and volume preserving embeddings of non-compact manifolds*, Trans. Amer. Math. Soc. **225** (1979), 403–414.

3. D. H. Gottlieb, *Fiber bundles and the Euler characteristic*, J. Differential Geom. **10** (1975), 39–48.

4. A. Haefliger, *Homotopy and integrability*, Lecture Notes in Math., vol. 197, Springer-Verlag, Berlin and New York, 1971, pp. 133–163.

5. ______, *Cohomology of Lie algebras and foliations*, Lecture Notes in Math., vol.652, Springer-Verlag, Berlin and New York, 1978, pp. 1–12.

6. ______, *Whitehead products and differential forms*, Lecture Notes in Math., vol. 652, Springer-Verlag, Berlin and New York, 1978, pp. 13–24.

7. S. Hurder, *Global invariants for measured foliations*, preprint, 1982.

8. D. McDuff, *Local homology of groups of volume preserving diffeomorphisms* I, Ann. Sci. Ecole Norm. Sup. (to appear).

9. ______, *Local homology of groups of volume preserving diffeomorphisms* III, preprint, 1982.

10. ______, *On groups of volume preserving diffeomorphisms and foliations with transverse volume form*, Proc. London Math. Soc. **43** (1981), 295–320.

11. G. Rousseau, *Difféomorphismes d'une variété symplectique non-compacte*, Comment. Math. Helv. **53** (1978), 622–633.

DEPARTMENT OF MATHEMATICS, STATE UNIVERSITY OF NEW YORK AT STONY BROOK, STONY BROOK, NEW YORK 11794