TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 291, Number 1, September 1985

# CORRECTION TO "VANISHING THEOREMS AND KÄHLERITY FOR STRONGLY PSEUDOCONVEX MANIFOLDS"

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It has been brought to my attention by N. Coltoiu that there was a gap in the proof of Theorem II in [3]. By using the same notations as in [3], the proof of our Theorem II can be corrected as follows.

THEOREM II. *Let* $(X, S)$ *be a strongly pseudoconvex manifold. If* $\dim S = 1$, *then* $X$ *is Kählerian. In particular, any strongly pseudoconvex surface is Kählerian.*

First of all the following result is needed.

LEMMA [1]. *Let* $S$ *be a compact* $\mathbf{C}$ *analytic space and let* $L$ *be a holomorphic line bundle on* $S$. *Let us assume that, for every positive dimensional subspace* $T$ *of* $S$, *there exist an integer* $n$ *and a nonzero holomorphic section of* $L^n \otimes O_T$ *which vanishes at some point of* $T$. *Then* $L$ *is positive.*

*Notations.* From now on, a *positive* line bundle (resp. *semipositive* line bundle) will be denoted by $L > 0$ (resp. $L \geqslant 0$).

PROOF OF THEOREM II. Without loss of generality, one can assume that $S$ is irreducible. So let $x$ be a point on $S$ and let $\pi\colon \hat{X} \to X$ be the blowing up of $X$ at $x$ inducing a biholomorphism $\hat{X} \setminus D \simeq X \setminus \{x\}$.

Let $T$ be the strict transform of $S$ under $\pi$; it is clear the $\hat{X}$ is a strongly pseudoconvex manifold with its exceptional set $\hat{S} = D \cup T$.

CLAIM. There exists a positive line bundle $L$ on $\hat{X} \setminus D$.

In fact, let $L_1 := [D]$ be the line bundle determined by $D$. Since $\dim T = 1$, the Lemma above tells us that $L_1 | T > 0$. In view of the compactness of $T$, one can find a relative compact neighborhood $V$ of $T$ in $\hat{X}$ such that

$(*)$ $L_1 > 0$ on $V$ and by construction $L_1 \geqslant 0$ on $\hat{X} \setminus D$.

Since $\hat{X}$ is strongly pseudoconvex, one can find a line bundle $L_2$ on $\hat{X}$ such that

$(**)$ $L_2 > 0$ on $\hat{X} \setminus \hat{S}$ and $L_2 \geqslant 0$ on $\hat{X}$.

In view of $(*)$ and $(**)$, it follows that, for some $N \gg 0$, the line bundle $L := L_1 \otimes L_2^N > 0$ on $(\hat{X} \setminus \hat{S}) \cup V \supset \hat{X} \setminus D$. Hence our claim is proved.

Consequently, $\hat{X} \setminus D \simeq X \setminus \{x\}$ is Kählerian. The result in [2] tells us that $X$ itself is Kählerian. Q.E.D.

## References

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