

CORRECTION TO "DIFFERENTIAL IDENTITIES IN PRIME RINGS WITH INVOLUTION"

CHARLES LANSKI

An example of Chuang [1] shows that the main results of [2] are false as stated. The purpose of this note is to state the correct versions of these theorems. We shall use the notation in [2], and all references to results are from that paper. We begin by noting that all of the results in [2] before Theorem 4 are correct as stated, and that the correction needed in Theorem 4 requires a subsequent change in Theorem 7 and in Theorem 9. All other results in the paper are correct.

The statement of Theorem 4 concerns a linear G^* -DI f , all of whose exponents come from W , the ordered collection of k -tuples of outer derivations which are independent modulo the inner derivations. For any exponent w appearing in f and coming from W , let f_w be the sum of all monomials in f with exponent w . The error in the proof of Theorem 4 is the assumption that if $f_w(x, y)$ is a G^* -PI for R , then $f_w(x^w, y^w)$ is also an identify for R . This is true when no involution is present, or equivalently, when y does not appear in f . However, given an exponent w appearing in f , a relation between r and r^* will not in general hold for r^w and $(r^*)^w$, unless $*$ commutes in $\text{End}(R)$ with w . Thus the induction used in Theorem 4 fails. The most important feature of Theorem 4 can be salvaged, using essentially the proof given.

For any w coming from $(d_1, \dots, d_k) \in W$, let k be the length of w . If $f \in F$ is linear and has all its exponents coming from W , an exponent w appearing in f is said to be of longest length if no other exponent of f has longer length. The conclusion of Theorem 4 is correct for all exponents of longest length, and the following is what the statement of the theorem should be.

THEOREM 4. *Let R be a prime ring with $*$, and let $f \in F$ be linear and have all its exponents coming from W , so that $f = \sum_h \sum_i a_{hi} x^h b_{hi} + \sum_k \sum_j c_{kj} y^k d_{kj}$ with all h and k coming from W and all coefficients in N . Suppose that for some nonzero ideal I of R , $f(I) = 0$. Then for each exponent w appearing in f and of longest length, $f_w(x) = \sum_i a_{wi} x b_{wi} + \sum_j c_{wj} y d_{wj}$ is a G^* -PI for R . In addition, if no y appears in f , or if each exponent appearing in f commutes with $*$ in $\text{End}(R)$, then $f_w(x)$ is a G^* -PI for R for every exponent w appearing in f .*

The proof proceeds as in [2], except that one uses induction on the longest length of exponents appearing in f . One may still assume that R satisfies a GPI by Theorem 1, and if 0 is the longest length then $f = f_1$ is a G^* -PI for R . As in [2], the expression $g(x) = f(cx) - cf(x)$ is a linear G^* -DI which contains no basis monomial appearing in f , and has its exponents of longest length at most

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one less than the longest length for f . Thus induction can be applied to g . In the case that the longest length for f is 1, $f = f_1(x, y) + \sum f_d(x^d, y^d)$, $g = \sum c^d f_d(x)$, and the Vandermonde type argument given in [2] shows that each $f_d(x)$ is a G^* -PI for R . For the general case, let w_1 be any exponent of f of longest length, and let w_1 come from $(d_1, m_2, \dots, m_k) \in W$. Write $w_1 = d_1 v$ where v comes from $(m_2, \dots, m_k)$. As in [2], by induction on k , $g_v(x)$ is a G^* -PI for R , and as in [2] one sees that $g_v(x) = \sum q_s c^{d_s} f_{w_s}(x)$ where w_s represents any exponent of f of length k which comes from a k -tuple having some d_s inserted in the appropriate place in the ordered $k-1$ tuple $(m_2, \dots, m_k)$. Also, q_s counts the number of occurrences of d_s in the k -tuple from which w_s comes. Since the collection of d_s appearing is independent modulo the inner derivations, the Vandermonde type argument shows again that each f_{w_s} is a G^* -PI for R , so in particular, $f_{w_1}(x)$ is.

To see how Theorem 7 needs to be changed in light of the change to Theorem 4, we recall that for $f \in F$ which is multilinear and homogeneous of degree n , and having all exponents coming from W , $W(f)$ is the set of all n -tuples $\bar{w} = (w_1, \dots, w_n)$ for which there is a monomial in f having each w_i as the exponent of x_i , or y_i . Then for any $\bar{w} \in W(f)$, $f_{\bar{w}}(x_1^{w_1}, \dots, x_n^{w_n}, y_1^{w_1}, \dots, y_n^{w_n})$ is the sum of all such monomials. Theorem 7 asserts that if f is a G^* -DI for an ideal I , then each $f_{\bar{w}}(x_1, \dots, x_n, y_1, \dots, y_n)$ is a G^* -PI for R . Using the correct statement of Theorem 4 above requires a restriction on which $\bar{w} \in W(f)$ one can use. Call $\bar{w} \in W(f)$ *special* if after some reordering of subscripts, the length of w_1 is maximal among the lengths of exponents in f appearing with either x_1 or y_1 , the length of w_2 is maximal among the lengths of exponents of either x_2 or y_2 , appearing in any monomial in which the exponent of x_1 , or of y_1 , is w_1 , and in general, the length of w_i is maximal among the lengths of exponents of x_i or y_i which appear in monomials for which $(w_1, \dots, w_{i-1})$ is the exponent sequence of the variables with subscript smaller than i . The proof of Theorem 7 is valid for all $\bar{w} \in W(f)$ which are special, and the following is the correct statement.

THEOREM 7. *Let R be a prime ring with involution, $*$, and let $f \in F$ be multilinear and homogeneous of degree n with all exponents coming from W and all subscripts of variables in $\{1, 2, \dots, n\}$. For any special $\bar{w} = (w_1, \dots, w_n) \in W(f)$, let $f_{\bar{w}}(x_1^{w_1}, \dots, x_n^{w_n}, y_1^{w_1}, \dots, y_n^{w_n})$ denote the sum of all monomials in f in which x_i or y_i appears with exponent w_i . If f is a G^* -DI for some nonzero ideal I of R , then $f_{\bar{w}}(x_1, \dots, x_n, y_1, \dots, y_n)$ is a G^* -PI for R , and R satisfies a GPI, unless $f = 0$ in F . Furthermore, the same conclusion holds for every $\bar{w} \in W(f)$ if either no y appears in f , or if each exponent appearing in f commutes with $*$ in $\text{End}(R)$.*

We note for Theorem 7 that in the case when every exponent appearing in f is either a derivation or is 1, then an exponent sequence is special if it contains a maximal number of derivations, although other sequences may be special. For example, if $W(f)$ consists of the sequences $(d, 1, d)$, $(1, d, 1)$, and $(h, 1, 1)$, then each would be special. Finally, it is important to observe that the applications we have made of Theorem 4 and Theorem 7 [3 and 4] are valid as given, using the corrected versions of these two theorems as they appear here.

The last correction needed in [2] is to change the hypothesis of Theorem 9 to assume that for each i , h_i is inner on Q exactly when k_i is inner on Q . With this modification and the comments above, the proof as given in [2] holds.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF SOUTHERN CALIFORNIA, LOS ANGELES, CALIFORNIA 90089-1113