A PLANCHEREL FORMULA FOR PARABOLIC SUBGROUPS

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ABSTRACT. We obtain explicit Plancherel formulas for the parabolic subgroups P of p-adic unitary groups which fix one dimensional isotropic subspaces. By means of certain limits of difference operators (called strong derivatives), we construct a Dixmier-Pukanszky operator which compensates for the nonunimodularity of the group P.

Then, we compute the Plancherel formula of $N \cdot A$, where N is the nilradical of P and $A = Q'_p$, the multiplicative group of nonzero p-adic numbers, by formulating a p-adic change of variable formula and using the strong derivative.

0. INTRODUCTION

Let p be a prime number not equal to 2. $\mathbf{Q}_p$ denotes the field of p-adic numbers with p-adic norm $|\cdot|_p$.

In this paper we obtain the Plancherel formula of parabolic subgroups P of p-adic unitary groups where the center of the nilradical has dimension one. To compensate for the nonunimodularity of the group we construct a pseudo-differential operator from a certain class of differential operators "strong derivatives" and the Plancherel formula turns out to take the form (Theorem 1.2)

$$f(1P) = c_1 \int_{\widehat{S}_1} \operatorname{tr} \pi_{1,\sigma} (\widetilde{D} f) \, d\hat{\mu}_{S_1}(\sigma)$$
$$+ c_2 \int_{\widehat{S}_1} \operatorname{tr} \pi_{n,\sigma} (\widetilde{D} f) \, d\hat{\mu}_{S_2}(\sigma).$$

We also compute the Plancherel formula (Theorem 3.1) of $N \cdot A$, where N is the nilradical of P and $A = \mathbf{Q}'_p$, the multiplicative group of nonzero p-adic numbers, by formulating a p-adic change of variable formula (Proposition 2.4) and by using the string derivative

$$\psi(1_{N \cdot A}) = T_1 \left(\underline{D} \psi + p T_p \left(\underline{D} \psi \right) + T_n \left(\underline{D} \psi \right) + p T_{sp} \left(\underline{D} \psi \right) \right)$$

where $T_\lambda(\psi)$ is the trace of an induced representation $\pi_\lambda(\psi)$.

I. Plancherel formula for a parabolic subgroup P_E
of p-adic unitary groups with $\dim E = 1$

1. Preliminaries—strong derivative for a Dixmier-Pukanszky operator. For the Plancherel formula for nonunimodular groups, we define a Dixmier-Pukanszky operator to compensate for the nonunimodularity of the group. In the p-adic case, some power of the strong derivative plays a role.

First, we introduce the definition of the strong derivative and its important properties. We follow Onneweer's paper [16].

Let Γ denote the character group $\widehat{\mathbf{Q}}_p$ of $\mathbf{Q}_p$. It contains a distinguished character χ_0 such that $\chi_0|_{B(0,0)} = 1$ and $\chi_0|_{B(0,-1)} \neq 1$, where $B(\alpha, n)$ is the ball centered at α with radius $1/p^n$. Then $\Gamma = \{\chi_y : y \in \mathbf{Q}_p\}$ where $\chi_y(x) = \chi_0(yz)$ for all $x \in \mathbf{Q}_p$. By dt we denote Haar measure on $\mathbf{Q}_p$, normalized so that the measure $B(0, 0)$ is one. We mean also by $\|\chi\|$ the usual norm $|y|_p$ of y if $\chi = \chi_y$.

Definition 1.1. Let $L^1_{\text{loc}}(\mathbf{Q}_p) = \{f : \mathbf{Q}_p \to C | f \text{ is locally } L^1\}$. For $m = 1, 2, 3, \ldots$, define $E_m f : \mathbf{Q}_p \to \mathbf{C}$ by

$$E_m f(x) = \sum_{l=-\infty}^{m-1} \left(p^{l+1} - p^l\right)\left(f(x) - \Delta_l * f(x)\right)$$

where

$$\Delta_l(x) = \begin{cases} p^l & \text{for } x \in B(0, l), \\ 0 & \text{otherwise}, \end{cases}$$

and $\Delta_l * f(x) = p^l \int_{B(x,l)} f(t)\, dt$ is the convolution which is the average of f over $B(x, l)$.

If $\lim_{m\to\infty} E_m f(x)$ exists for $x \in \mathbf{Q}_p$, then this limit is called the pointwise derivative of f at x and is denoted by $f^{[1]}(x)$.

If $f \in C(\mathbf{Q}_p)$, $\{q : \mathbf{Q}_p \to \mathbf{C} | g \text{ is continuous}\}$, and if the limit is in $C(\mathbf{Q}_p)$, then the limit is called the strong derivative of f in $C(\mathbf{Q}_p)$ and is denoted by $D^{[1]} f$.

Remark. It can be checked that locally constant functions with compact support have strong derivatives.

The following theorem gives an important property of the strong derivative.

Theorem 1.1 (Onneweer [16]). (1) *If $\chi \in \Gamma$ and $x \in \mathbf{Q}_p$, then $\chi^{[1]}$ exists and $\chi^{[1]}(x) = \|\chi\|(x)$.*

(2) *Every $\chi \in \Gamma$ is strongly differentiable in $C(\mathbf{Q}_p)$, the space of all continuous functions.*

(3) *For $f \in L^1(\mathbf{Q}_p)$ and $n = 1, 2, 3, \ldots$,*

$$(E_n f)\hat{}(\chi) = \begin{cases} \|\chi\|\hat{f}(\chi) & \text{if } \chi \in \Gamma_n, \\ p^n \hat{f}(\chi) & \text{if } \chi \notin \Gamma_n, \end{cases}$$

where $\Gamma_n = \{\chi = \chi_y \in \Gamma | y \in B(0, -n)\}$ *and* $\char`^$ *is the Fourier transform defined by* $\hat{f}(\chi) = \int_{\mathbf{Q}_p} f(t)\overline{\chi(t)}\, dt$.

Corollary (Onneweer [16]). *Let* $D(D^{|1|})$ *denote the domain of* $D^{|1|}$. *Then for* $f \in D(D^{|1|})$, $(D^{|1|}f)^\char`^(\chi) = \|\chi\|\hat{f}(\chi)$, *for all* $x \in \Gamma$.

This property implies that the strong derivative can be characterized as a multiplication operator $M_{|y|_p}$ by $|y|_p$ via Fourier transform, i.e. $D^{|1|} = \wedge^{-1} \cdot M_{|y|_p} \cdot \wedge$. Clearly, $D^{|1|}$ is positive, selfadjoint, invertible, since $|y|_p$ is positive, real-valued, and takes zero only on a set of measure zero.

2. Plancherel formula for parabolic subgroups P_e of p-adic unitary groups with $\dim E = 1$.

Let p be a rational prime number not equal to 2 and let $\mathbf{Q}_p$ denote the p-adic number field. Let (W, h) be a Hermitian vector space over $\widetilde{\mathbf{Q}}_p$ relative to $\mapsto \overline{a}$. Here $\widetilde{\mathbf{Q}}_p = \mathbf{Q}_p(\sqrt{p})$ is a quadratic extension by $\sqrt{p}$, $a \mapsto \overline{a}$ is the nontrivial element of the Galois group of $\widetilde{\mathbf{Q}}_p$ over $\mathbf{Q}_p$.

Let $E \subseteq W$ be an isotropic subspace, i.e. $h(E, E) = \{0\}$. It is known that there is a totally isotropic subspace $E' \subseteq W$ such that $\dim E = \dim E'$ and h is nondegenerate. We write $W = V + E' + E$ so that $V = (E' + E)^\perp$ relative to h. If we denote by $U(W, h)$ the group of unitary operators on W with respect to the Hermitian form h, then $P_E = \{b \in U(W, h) | bE \subseteq E\}$ is a maximal parabolic subgroup of $U(W, h)$ and this construction gives all such maximal parabolics.

The structure of such p-adic parabolic subgroups is essentially the same as in the real case; for $n = \dim V$ and $S = \dim E$ the following is true:

$$P_E = (Z \cdot X) \cdot \big(GL(E) \times U(V, h|_{V \times V})\big)$$

where $N = Z \cdot X$ is the nilradical, $L = GL(E) \times U(V, h|_{V \times V})$ is the Levi component, and $Z = \{z \in \mathbf{Q}_p^{s \times s}, \text{ an } s \times s \text{ matrix}: {}^t\overline{z} = -z\}$, $X = \mathbf{Q}_p^{s \times n} = \{s \times n$ matrices over $\widetilde{\mathbf{Q}}_p\}$. The group multiplication of P is

$$(z_1, x_1, \gamma_1, g_1)(z_2, x_2, \gamma_2, g_2)$$
$$\big(z_1 + \gamma_1 z_2 {}^t\overline{\gamma}_1 + \operatorname{Im} H(x_1, \gamma_1 x_2 g_1^{-1}), x_1 + \gamma_2 x_2 g_1^{-1}, \gamma_1\gamma_2, g_1 g_2\big)$$

where $H(x', x'') = x'' M^{-1}{}^t\overline{x}'$ and $\operatorname{Im} H(x', x'') = \frac{1}{2}[H(x', x'') - H(x'', x')]$. By choosing a basis $\{v_1, \dots, v_n\}$ of V, $\{e'_1, \dots, e'_s\}$ of E' and $\{e_1, \dots, e_s\}$ of E such that $h(e'_i, e_j) = \delta_{ij}$, the matrix form of M_h of the Hermitian form h is written as

$$M_h = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{pmatrix} \quad \text{with } {}^t\overline{M} = M.$$

The following theorem can be proved in essentially the same way as the real case (Lipsman-Wolf [11]).

Theorem 1.2. *Let* $P = P_E$ *with* $\dim E = 1$. *Then*

$$f(1_P) = c_1 \int_{\widehat{S_1}} \text{tr } \pi_{1,\sigma}(\widetilde{D}f) \, d\hat\mu_{S_1}(\sigma) + c_2 \int_{\widehat{S_2}} \text{tr } \pi_{n,\sigma}(\widetilde{D}f) \, d\hat\mu_{S_2}(\sigma),$$

where c_i *are some real constants,* $S_i = U(\widetilde{\mathbf{Q}}_p, h_i) \times U(V, h|_{V \times V})$, $i = 1, 2$, h_1 *and* h_2 *are inequivalent Hermitian forms over* $\widetilde{\mathbf{Q}}_p$, *for example,* $h_1 = (1)$ *and* $h_2 = (\mu)$ *where* μ *is a unit element of* $\mathbf{Q}_p$ *which is not a square as an element of residue class field,* $\hat\mu_{S_1}$ *is the Plancherel measure on* $\widehat{S}_i$ *corresponding to a uniquely determined Haar measure* μ_{S_i} *according to the formula*

$$\int_L f(x) \, dx = \int_{S_i \backslash L} \int_{S_i} f(s_i x) \, ds_i \, d\overline{x}$$

where $d\overline{x}$ *is the Borel measure on* $S_i \backslash L$ *and* $\pi_{1,\sigma} = \text{Ind}_{NS_1}^P(\tilde\gamma_1 \times \sigma)$, $\sigma \in \widehat{S}_1$, $\pi_{n,\sigma} = \text{Ind}_{NS_2}^P(\tilde\gamma_n \times \sigma)$, $\sigma \in \widehat{S}_2$, $\tilde\gamma_1$ *and* $\tilde\gamma_2$ *are extensions of* γ_1 *and* γ_2, *representatives of* L-*orbits of* $\widehat{N}$. *Finally,* $\widetilde{D} = D^{n+1}$, *where* $D = |1|$, *the* $(n+1)$*th power of the strong derivative.*

Now, to obtain a more explicit formula for our parabolic subgroup P, we have to compute p-adic L-orbits of Z, which will turn out to be equivalent to classifying nondegenerate Hermitian forms over $\widetilde{\mathbf{Q}}_p$, and we have to compute the corresponding stability groups. We treat this without any restrictions on $\dim E$.

3. p-adic L-orbits of Z and the L-stabilizer. In order to apply Mackey's little group method, we first find the L-orbits on Z.

L acts on Z by $(\gamma, g) \cdot z = \gamma z \gamma^s$. So the L-orbits are represented by $\{\sqrt{p}h \,|\, [h]$ is an equivalence class of Hermitian forms on $\widetilde{\mathbf{Q}}_p^s\}$. Thus we are going to classify Hermitian forms on $\widetilde{\mathbf{Q}}_p^s$ up to equivalence.

The following lemma can be proved easily.

Lemma 1.1. *Given a symmetric bilinear form* q *on* $\mathbf{Q}_p^{2s}$, *there is a Hermitian form* h *on* $\widetilde{\mathbf{Q}}_p^s$ *such that*

$$h(X, Y) = q(\widetilde{X}, \widetilde{Y}) + \frac{1}{\sqrt{p}} q(\sqrt{p}\widetilde{X}, \widetilde{Y})$$

if and only if $q(\sqrt{p}\widetilde{X}, \widetilde{Y}) + q(\widetilde{X}, \sqrt{p}, \widetilde{Y}) = 0$ *for all* X, $Y \in \overline{\mathbf{Q}}_p^s$, *where* $\widetilde{X}$ *and* $\widetilde{Y}$ *are elements of* $\mathbf{Q}_p^{2s}$ *corresponding to* X *and* Y *under the isomorphism* $\mathbf{Q}_p^{2s} \cong \mathbf{Q}_p(\sqrt{p})^s$ *and* $\sqrt{p}\widetilde{X}$ *is regarded as an operator* $\widetilde{X} \mapsto \sqrt{p}\widetilde{X}$ *on* $\mathbf{Q}_p^{2s}$ *operating on* $\widetilde{X}$.

Thus we only need to find the symmetric nondegenerate bilinear forms on $\widetilde{\mathbf{Q}}_p^s$, up to equivalence, satisfying the condition of Lemma 1.1.

Definition 1.2. A quadratic form q on V is said to be *isotropic* if $q(x) = 0$ for some $x \neq 0$ in V and is said to be an *anisotropic* otherwise.

Definition 1.3. A Hermitian form is said to be *anisotropic* if the quadratic form defined by the Hermitian form is anisotropic.

We are going to classify the anisotropic Hermitian forms first.

Lemma 1.2. (1) *Anisotropic nondegenerate Hermitian forms over* $\widetilde{\mathbf{Q}}_p$ *are at most two dimensional.*

(2) *There are only two equivalence classes of* 1-*dimensional ones; they are represented by* 1×1 *matrices* (1) *and* (u), *where* $u \in \mathbf{Q}_p$ *is a unit such that* u, *as an element of the residue class field, is not a square.*

(3) *There is only one equivalence class of* 2-*dimensional ones; it is represented by the* 2×2 *matrix* $\left(\begin{smallmatrix} 1 & 0 \\ 0 & -u \end{smallmatrix}\right)$.

Proof. By the theory of quadratic forms, we know that anisotropic nondegenerate quadratic forms are at most 4-dimensional (see [9]). Hence anisotropic nondegenerate Hermitian forms are at most 2-dimensional. So we get (1).

Let q be a quadratic form which gives rise to a Hermitian form. Then $\dim q$ is either 2 or 4. If $\dim q = 2$, write $q = \left(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}\right)$ with a, $b \in \mathbf{Q}_p$. The condition in Lemma 1.1 that $q(\sqrt{p}\widetilde{X}, \widetilde{Y}) + q(\widetilde{X}, \sqrt{p}, \widetilde{Y}) = 0$ for all $\widetilde{X}$, $\widetilde{Y} \in \mathbf{Q}_p^{2s}$ means ${}^t\sqrt{p}\,q + q\sqrt{p} = 0$, where ${}^t\sqrt{p}$ means the transpose of the operator $\sqrt{p}$. So

$$\begin{pmatrix} 0 & 1 \\ p & 0 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} 0 & p \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

So $b = -ap$. Hence $q = \left(\begin{smallmatrix} a & 0 \\ 0 & -ap \end{smallmatrix}\right)$.

It is known that there are only six inequivalence classes of nondegenerate anisotropic quadratic forms over $\mathbf{Q}_p$, namely

$$\begin{pmatrix} 1 & \\ & -u \end{pmatrix}, \quad \begin{pmatrix} p & \\ & -pu \end{pmatrix}, \quad \begin{pmatrix} 1 & \\ & -p \end{pmatrix},$$
$$\begin{pmatrix} u & \\ & -pu \end{pmatrix}, \quad \begin{pmatrix} 1 & \\ & -pu \end{pmatrix}, \quad \begin{pmatrix} u & \\ & -p \end{pmatrix}.$$

We notice that among those only $\left(\begin{smallmatrix} 1 & \\ & -p \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} u & \\ & -pu \end{smallmatrix}\right)$ are of the form $\left(\begin{smallmatrix} a & 0 \\ 0 & -ap \end{smallmatrix}\right)$.

If $\dim q = 4$, again from the theory of quadratic forms over $\mathbf{Q}_p$, we know that there is only one class of nondegenerate anisotropic quadratic forms, namely

$$q = \begin{pmatrix} 1 & & & \\ & -u & & \\ & & -p & \\ & & & -up \end{pmatrix}.$$

Claim. The Hermitian form giving $q = \begin{pmatrix} 1 & \\ & -p \end{pmatrix}$ is $h = (1)$. The Hermitian form giving $q' = \begin{pmatrix} u & \\ & -up \end{pmatrix}$ is $h' = (u)$. The Hermitian form giving

$$q'' = \begin{pmatrix} 1 & & & \\ & -u & & \\ & & -p & \\ & & & up \end{pmatrix}$$

is $h'' = \begin{pmatrix} 1 & \\ & -u \end{pmatrix}$.

Proof of Claim. Note $h(X, X) = q(\widetilde{X}, \widetilde{X})$ because the skew-symmetric part gives zero. Let $X = x + \sqrt{p}y$, $x, y \in \mathbf{Q}_p^s$. Then

$$h(X, X) = \overline{X}(1)X = \overline{X}X = (x - \sqrt{p}y)(x + \sqrt{p}y) = (x, y) \begin{pmatrix} 1 & \\ & -p \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= q((x, y), (x, y)) = q(\widetilde{X}, \widetilde{X}).$$

So $h = (1)$ is the Hermitian form giving $q = \begin{pmatrix} 1 & \\ & -p \end{pmatrix}$. Similarly, for $q' = \begin{pmatrix} u & \\ & -up \end{pmatrix}$ and

$$q'' = \begin{pmatrix} 1 & & & \\ & -u & & \\ & & -p & \\ & & & up \end{pmatrix}.$$

This proves the claim.

Next, we have to show the following: If q and q' are given by Hermitian forms h and h', respectively, and if q and q' are not equivalent, then h and h' are not equivalent.

To prove this suppose that h and h' are equivalent. Then there exists $A \in GL(s, \widetilde{\mathbf{Q}}_p)$ such that $h'(AX, AY) = h(X, Y)$ for all $X, Y \in \widetilde{\mathbf{Q}}_p^s$. Now,

$$h'(AX, AY) = q'(\widetilde{A}\widetilde{X}, \widetilde{A}\widetilde{Y}) + \frac{1}{\sqrt{p}}q'(\widetilde{A}\widetilde{X}, \widetilde{A}\widetilde{Y}),$$

where $\widetilde{A}$ is the operator $\in GL(2s, \mathbf{Q}_p)$ determined by A. Also,

$$h(x, Y) = q(\widetilde{X}, \widetilde{Y}) + \frac{1}{\sqrt{p}}q(\sqrt{p}\widetilde{X}, \widetilde{Y}).$$

Comparing the $\mathbf{Q}_p$ components we get $q'(\widetilde{A}\widetilde{X}, \widetilde{A}\widetilde{Y}) = q(\widetilde{X}, \widetilde{Y})$. So q and q' are inequivalent. Therefore now we know that the 1-dimensional case (1) and (u) are inequivalent.

Finally, we have to show that there are no more inequivalent classes.

We first consider the 1-dimensional case. Let $h = (1)$ and $h' = (u)$. Suppose that there exists $h' = (a)$ with $a \in \mathbf{Q}_p$ such that $h'' \nsim h'$ where $\sim$ means equivalent and $\nsim$ means not equivalent. This implies that there is no $\alpha \in \mathbf{Q}_p(\sqrt{p})$ such that $\bar{\alpha}\alpha u = a$, i.e., $a \neq x^2 - py^2$ for any $x, y \in \mathbf{Q}_p$ and $a \neq u(x^2 - py^2)$ for any $x, y \in \mathbf{Q}_p$. Now, let q'' be the quadratic form given

by h''. Then $q'' = \begin{pmatrix} a & \\ & -ap \end{pmatrix}$. Since q'' is anisotropic and nondegenerate, either $q'' \sim \begin{pmatrix} 1 & \\ & -p \end{pmatrix}$ or $q'' \sim \begin{pmatrix} u & \\ & -up \end{pmatrix}$. If $q'' \sim \begin{pmatrix} 1 & \\ & -p \end{pmatrix}$, there must be $c \in GL(2, \mathbf{Q}_p)$ such that ${}^t C \begin{pmatrix} 1 & \\ & -p \end{pmatrix} C = \begin{pmatrix} a & \\ & -ap \end{pmatrix}$. Write $C = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$. Then

$$ {}^t C \begin{pmatrix} 1 & \\ & -p \end{pmatrix} C = \begin{pmatrix} x^2 - pz^2 & xy - pzw \\ xy - pzw & y^2 - pw^2 \end{pmatrix}. $$

So ${}^t C \begin{pmatrix} 1 & \\ & -p \end{pmatrix} C = \begin{pmatrix} a & \\ & -ap \end{pmatrix}$ forces that $x^2 - pz^2 = a$ in particular, which is impossible by the assumption that $h'' \not\sim h$. By a similar argument, there is no h'' such that $h'' \not\sim h'$.

Now, to complete the classification we have to consider isotropic ones. We recall that if h is a 2-dimensional nondegenerate isotropic Hermitian form over $\widetilde{\mathbf{Q}}_p$, then there exists a basis with respect to which h has its matrix representation $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Here is the result of classification of nondegenerate Hermitian forms over $\widetilde{\mathbf{Q}}_p$.

Proposition 1.1. *Any nondegenerate Hermitian form over $\widetilde{\mathbf{Q}}_p$ is of the form*

$$ \begin{pmatrix} 1 & & & & & \\ & -1 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & -1 & \\ & & & & & A \end{pmatrix} \quad or \quad \begin{pmatrix} 1 & & & & & \\ & -1 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & 1 \\ & & & & & & -1 \end{pmatrix} $$

with A anisotropic. Hence, the number of equivalence classes of s-dimensional nondegenerate Hermitian forms over $\widetilde{\mathbf{Q}}_p$ is always 2.

In the 2-dimensional case, the argument is the same. So we omit this. Hence we have proved Lemma 1.2.

The translation of Proposition 1.1 over to a result on the L-orbits of Z is as follows:

Proposition 1.2. *For any positive integer s there are precisely two generic L-orbits of Z; they are represented by $\sqrt{p}h_1$, $\sqrt{p}h_2$ where h_1 and h_2 are inequivalent nondegenerate Hermitian forms over $\widetilde{\mathbf{Q}}_p$. The L-stabilizer S_i of $\sqrt{p}h_i$ is given by $S_i = U(\widetilde{\mathbf{Q}}_p^2, h_i) \times U(V, h|_{V \times V})$.*

4. Dixmier-Pukanszky operator for P_E with $\dim E = 1$. We shall show that a certain power of the strong derivative will give a Dixmier-Pukanszky operator for a parabolic subgroup P_E of $U(W, h)$ with $\dim E = 1$.

For this parabolic group the modular function δ_P of P is $\delta_P(z, x, a, m) = |a|_p^{2(1+n)}$, where $A = \mathbf{Q}_p^s = \{\text{nonzero } p\text{-adic numbers}\}$, since the action of A on N is $a \cdot (z, x) = (a^2 z, ax)$ and only A-part of L has significance for δ_P.

Now, let $\tilde{D} = D^{n+1}$ as in §2. Then this $\tilde{D}$ clearly satisfies the criterion for being a Dixmier-Pukanszky operator, namely the criterion $x \cdot \tilde{D} = \delta_p(x)\tilde{D}$ for all $x \in L$ where $x \cdot \tilde{D}f(y) = \tilde{D}f(xyx^{-1})$.

Hence $\tilde{D}$ plays the role of Dixmier-Pukanszky operator and the proof of the theorem is complete.

II. Plancherel formula for $N \cdot A$

In this section we compute directly the Plancherel formula for $N \cdot A$.

Let (W, h) be a Hermitian vector space over $\tilde{\mathbf{Q}}_p$ relative to $a \mapsto \bar{a}$ and consider the case that the dimension of the center of N is 1 as before. To make the comparison with the real case clearer, we choose h so that

$$M_h = \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{pmatrix}.$$

Hence our group is $N \cdot A$ with $N = Z \cdot X$, $Z = \{\sqrt{p}z | z \in \mathbf{Q}_p\}$, $X = \tilde{\mathbf{Q}}_p^n$, $A = \mathbf{Q}_p^n$, $A = \mathbf{Q}_p^s$ = the nonzero p-adic numbers and p is a prime number not equal to 2. We identify the group $N \cdot A$ with $\mathbf{Q}_p \times \mathbf{Q}_p^n \times \mathbf{Q}_p^n \times \mathbf{Q}_p^s$ with group multiplication:

$$(z, y, x, a)(z', y', x', a') = (z + a^2 z' + ay'x - ax'y, y + ay', x + ax', aa').$$

The computation technique of the computation of the Plancherel formula for this particular group is mostly the same as the real case (see [4]). But in the p-adic case it is necessary to find a p-adic change of variable formula.

1. p-adic change of variable formula. A computation shows the following.

Proposition 2.1. *Let* $\alpha \in \mathbf{Q}_p$ *and* $n \in \mathbf{Z}$ *be such that* $0 \notin B(\alpha, n)$ *where* $B(\alpha, n) = \{x \in \mathbf{Q}_p | |\alpha - x|_p \le 1/p^n\}$. *Let* $S: \mathbf{Q}_p \to \mathbf{Q}_p$ *be the square function, i.e.,* $S(x) = x^2$. *Then* $S(B(\alpha, n)) = B(\alpha^2 n + k)$, *where* $|\alpha|_p = 1/pk$.

Now, the idea of the proof of the p-adic change of variable formula is that first we observe that the whole $\mathbf{Q}_p$ is a disjoint union of shells centered at 0, i.e. Shell$(0, n) = \{x \in \mathbf{Q}_p | |x|_p = 1/p^n\}$ and observe that each shell is a disjoint union of $p - 1$ balls of radius $1/p^{n+1}$. Then we obtain the formula on shells and eventually obtain the formula on $\mathbf{Q}_p$ by dominant convergence theorem.

The following lemma and the proposition are easily obtained by computation.

Lemma 2.1. *The square function* S *is one-one on* $B(\alpha, n)$ *if and only if* $0 \notin B(\alpha, n)$.

We need also the following technical proposition.

Proposition 2.2. *For* $p \ne 2$, *let* $B(\alpha, n) \subseteq \mathbf{Q}_p$ *be such that* $0 \notin B(\alpha, n)$. *Then*

$$\int_{B(\alpha, n)} 1_{B(\beta, k)}(x^2) |x|_p \, dx = \int_{S(B(\alpha, n))} 1_{B(\beta, k)}(x) \, cx$$

for any $B(\beta, k) \subseteq \mathbf{Q}_p$. *Here* $S: \mathbf{Q}_p \to \mathbf{Q}_p$ *is defined by* $S(x) = x^2$ *as before and* 1_A *is a characteristic function on* A, *i.e.*,

$$1_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise}. \end{cases}$$

The next step is to obtain the formula on shells. We recall that each shell is a disjoint union of balls. We will make use of this fact fully later.

Lemma 2.2. *Let* $p \neq 2$.

$$S\big(B(ip^{-n}, -n+1)\big) = S\big(B((p-i)p^{-n} - n + 1)\big)$$

for all $i = 1, \ldots, (p-1)/2$. *It can be proven by direct computation. Here* i *is the intermediate form of the p-adic change of variable formula.*

Proposition 2.3.

$$\int_{\text{Shell}(0,0)} f(x^2) |x|_p \, dx = 2 \int_{S(\text{Shell}(0,0))} f(x) \, dx$$

for all $f \in C_c^\infty(\mathbf{Q}_p) = \{f: \mathbf{Q}_p \to \mathbf{C} \,|\, f$ *is locally constant and has compact support*$\}$, *where* $\text{Shell}(0, 0) = \{x \in \mathbf{Q}_p \,|\, |x|_p = 1\}$.

Proof. Note: We can write $\text{Shell}(0, 0)$ as the disjoint union of balls, $\text{Shell}(0, 0) = B(1, 1) \cup \cdots \cup B(p-1, 1)$. Also, since f is compactly supported, $\text{supp} f \subseteq B(0, -n)$ for some $n \in \mathbf{Z}$ and since f is locally constant, f is constant on balls of radius $1/p^k$ for some $k \in \mathbf{Z}$. We may assume that $n > -k$. Now

$$\int_{\text{Shell}(0,0)} f(x^2) |x|_p \, dx = \sum_{i=1}^{p-1} \int_{B(i,1)} f(x^2) |x|_p \, dx$$

$$= \sum_{i=1}^{p-1} \int_{B(i,1)} 1_{B(i^2,1)} f(x^2) |x|_p \, dx, \quad \text{by Proposition 2.1}.$$

But by using Proposition 2.2 and Lemma 2.2 we can show

$$\int_{B(i,1)} 1_{B(i^2,1)} f(x^2) |x|_p \, dx = \int_{B(p-i,1)} 1_{B((p-i)^2,1)} f(x^2) |x|_p \, dx$$

for all $i = 1, \ldots, (p-1)/2$. So we have

$$\int_{\text{Shell}(0,0)} f(x^2) |x|_p \, dx = 2 \left[\sum_{i=1}^{(p-1)/2} \int_{S(B(i,1))} 1_{B(i^2,1)} f(x) \, dx \right]$$

$$= 2 \int_{S(\text{Shell}(0,0))} f(x) \, dx.$$

Proposition 2.3 is also true for general shells. In fact we get the following corollary.

Corollary. $\int_{\text{Shell}(0,\,-l)} f(x^2)|x|_p\,dx = 2\int_{S(\text{Shell}(0,\,-l))} f(x)\,dx$ *for all* $f \in C_c^\infty(\mathbf{Q}_p)$
and for all $l \in \mathbf{Z}$ *where* $\text{Shell}(0,-l) = \{x \in \mathbf{Q}_p | |x|_p = p^l\}$. *This can be proved by using Proposition 2.3 and Lemma 2.2. Although the proof is easy but lengthy, we omit it.*

With the dominated convergence theorem, the corollary to Proposition 2.3 and Lemma 2.2 we get the following p-adic change of variable formula.

Proposition 2.4. *Let* $p \neq 2$ *be a rational prime number. Then*

$$\int_{\mathbf{Q}_p} f\left(x^2\right)|x|_p\,dx = 2\int_{S(\mathbf{Q}_p)} f(x)\,dx$$

for all $f \in C_c^\infty(\mathbf{Q}_p)$. *Here* $S(x) = x^2$.

2. Plancherel formula for $N \cdot A$ over a p-adic number field.

We first need to describe the generic irreducible unitary representation of $N\cdot A$ i.e., those which contribute to the Plancherel formula. The center of N is $Z = \mathbf{Q}_p$ (recall we identified $N \cdot A$ with $\mathbf{Q}_p \times \mathbf{Q}_p^n \times \mathbf{Q}_p^n \times \mathbf{Q}_p^s$ at the beginning of this chapter). Let $\mathbf{Z}$ be the Lie algebra of Z and $\mathbf{Z}^s$ be the dual space of $\mathbf{Z}$ as $\mathbf{Q}_p$-vector space.

Now, for each $\lambda \in \mathbf{Z}^s$ which is nonzero, there exists an irreducible unitary representation class $[\gamma_\lambda] \in \widehat{N}$, the set of all equivalence classes of irreducible unitary representations of N, uniquely determined by the equation $\gamma_\lambda(zn) = \lambda(z)\gamma_\lambda(n)$ for $z \in Z$ and $n \in N$ (see for details [20, 13, 4]). In fact let $\mathbf{N} =$ Lie algebra of N. We choose a polarization $\mathbf{U}$ at $\lambda \in \mathbf{N}^s$, i.e., $\mathbf{U}$ is a maximal isotropic subspace for the bilinear form B_λ defined by λ as $B_\lambda(n_1, n_2) = \lambda([n_1, n_2])$ for $n_1, n_2 \in \mathbf{N}$. Let χ_λ be a character of $U = \exp\mathbf{U}$ by $\chi_\lambda(u) = \chi_0(\lambda(z))$, where z is the component of u from the center and χ_0 is the fixed character of $\mathbf{Q}_p$ as usual. Then $\gamma_\lambda = \text{Ind}_U^N(\chi_\lambda)$. We remark that the reason why it is sufficient to consider $\lambda \in \mathbf{Z}^s$ instead of $\lambda \in \mathbf{N}^s$ is that in our particular case, for any $\lambda \in \mathbf{N}^s$, $\lambda([z, y]) = \lambda([z, x]) = \lambda([x, x']) = \lambda([y, y']) = 0$ for any z, y, y', x, and x'. So we only need to consider $\lambda \in \mathbf{Z}^s$ and extend it trivially to $\mathbf{N}$.

We recall that $[\gamma_\lambda] \neq [\gamma_{\lambda'}]$ if $\lambda \neq \lambda'$ and these are the generic representation of N. The generic representations of $N \cdot A$ are the induced representations induced from those $[\gamma_\lambda]$ such that γ_λ is a representative of A-orbit on $\widehat{N}$. A acts on $[\gamma_\lambda]$ by $a \cdot [\gamma_\lambda] = [\gamma_{a^2\lambda}]$. So we use only four equivalence classes due to the fact that $\mathbf{Q}_p^s$ has exactly four cosets of the subgroup of elements which are squares of some other elements (see [3]).

$N \cdot A$ has the right Haar measure. We denote it by

$$d(z, y, x, a) = \frac{1}{|a|_p}\,dx\,dy\,dx\,da,$$

where dz, dy, dx, da are invariant measures on $\mathbf{Q}_p$, $\mathbf{Q}_p^n$, $\mathbf{Q}_p^n$, and $\mathbf{Q}_p$ respectively with dz and da normalized so that the measure of the unit ball is one.

Theorem 3.1. *Denote* $\underline{D} = D^{n+1}/^{n+1}\sqrt{2}$, *where* D *is the strong derivative (Chapter* I, *part* I*). Let* u *be a unit element in* $\mathbf{Q}_p$ *that does not represent a square in the residue class field. (For example if* $p = 3$, $u = 2$, *and if* $p = 5$, u *is either* 2 *or* 3*.) Let* $\pi_\lambda = \mathrm{Ind}_N^{N \cdot A}(\gamma_\lambda)$ *for* $\lambda = 1, p, u, up$*. Let* $E = \{\psi \in C_c^\infty(N \cdot A) | \psi$ *is a linear combination of* $\varphi * \varphi^s$ *with* $\varphi \in C_c^\infty(N \cdot A)\}$. *If* $\psi \in E$, *then* $\underline{D}\psi \in L^1(N \cdot A)$ *and* $\pi_\lambda(\underline{D}\psi)$ *is trace class for* $\lambda \neq 0$. *The Fourier inversion formula for* $N \cdot A$ *is*

$$\psi\left(1_{N \cdot A}\right) = T_1\left(\underline{D}\psi\right) + pT_p\left(\underline{D}\psi\right) + T_n\left(\underline{D}\psi\right) + pT_{np}\left(\underline{D}\psi\right)$$

for all $\psi \in E$, *where* $T_\lambda(\psi) =$ trace of $\pi_\lambda(\psi)$.

Comment. The space E is dense in $L^2(N \cdot A)$.

Proof of Theorem 3.1. Notice that every element f in the representation space of π_λ can be written as $f(z, y, x, a) = \chi_\lambda(z, y, 0, 1)f(0, 0, x, a)$ because $f(z, y, x, a) = f((z, y, 0, 1)(0, 0, x, a)) = \chi_\lambda(z, y, 0, 1)f(0, 0, x, a)$. Let $\varphi_f(x, a) = f(0, 0, x, a)$. Then

$$\varphi_f \in L^2\left(\mathbf{Q}_p^n \times \mathbf{Q}_p^s, \frac{1}{|a|_p} \, dx \, da\right).$$

Moreover, the map

$$L^2\left(\frac{N \cdot A}{U}, \chi_\lambda\right) \to L^2\left(\mathbf{Q}_p^n \times \mathbf{Q}_p^s, \frac{1}{|a|_p} \, dx \, da\right)$$

by $f \mapsto \varphi_f$ is unitary and intertwines π_λ and $\tilde{\pi}_\lambda$, where

$$\tilde{\pi}_\lambda(z, y, x, a) : L^2\left(\mathbf{Q}_p^n \times \mathbf{Q}_p^s, \frac{1}{|a|_p} \, dx \, da\right) \to L^2\left(\mathbf{Q}_p^n \times \mathbf{Q}_p^s, \frac{1}{|a|_p} \, dx \, da\right)$$

is defined by $\tilde{\pi}_\lambda(z, y, x, a)\varphi_f = \varphi_{\pi_\lambda(z, y, x, a)f}$. So we view π_λ as a representation on

$$L^2\left(\mathbf{Q}_p^n \times \mathbf{Q}_p^s, \frac{1}{|a|_p} \, dx \, da\right).$$

We use the same notation π_λ instead of $\tilde{\pi}_\lambda$. Then

$$\pi_\lambda\left((z', y', x', a')\varphi_f\right)(x, a) = f(a^2 z' + ax \cdot y', ay', x + ax', aa')$$
$$= f\left((a^2 z' + a^2 x' \cdot y' + 2ax \cdot y', ay', 0, 1)(0, 0, x + ax', aa')\right)$$
$$= \chi_\lambda(a^2 z' + a^2 x' \cdot y' + 2ax \cdot y')\varphi_f(x + ax', aa').$$

Now let $\psi \in C_c^\infty(N \cdot A)$. Then

$$(\pi_\lambda(\psi)\varphi_f)(x, a)$$
$$= \int_{N \cdot A} \psi(z', y', x', a')(\pi_\lambda(z', y', x', a')\varphi_f)(x, a)\frac{1}{|a'|_p}\,dz'\,dy'\,dx'\,da'$$
$$= \int_{N \cdot A} \psi(z', y', x', a')\chi_\lambda(a^2 z' + a^2 x' \cdot y' + 2ax \cdot y')$$
$$\cdot \varphi_f(x + ax', aa')\frac{1}{|a'|_p}\,dz'\,dy'\,dx'\,da'.$$

We claim that

$$(\pi_\lambda(\psi)\varphi_f)(x, a)$$
$$= \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} \hat{\psi}\left(\cdot, \cdot, \frac{x' - x}{a}, \frac{a'}{a}\right)(\lambda a^2, \lambda a(x' + x))\varphi_f(x', a')|a'|_p^{-n}\frac{1}{|a'|_p}\,dx'\,da'$$

where $\hat{\psi}$ is the Fourier transform of ψ in the first two variables.

Now let

$$K(x, a, x', a') = \hat{\psi}\left(\cdot, \cdot, \frac{x' - x}{a}, \frac{a'}{a}\right)(\lambda a^2, \lambda a(x' + x))\frac{|a|_p^{-n}}{|a'|_p}.$$

Then

$$(\pi_\lambda(\psi)\varphi_f)(x, a) = \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} K\left(x, a, \frac{x' - x}{a}, a'\right)\varphi_f(x', a')\,dx'\,da'.$$

Now if $\psi \in E$ in particular, $\pi_\lambda(\psi)$ is of trace class. Let $T_\lambda(\psi) = $ trace of $\pi_\lambda(\psi)$. Then

$$T_\lambda(\psi) = \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} K(x, a, \lambda, a)\,dx\,da$$
$$= \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} \hat{\psi}(\cdot, \cdot, 0, 1)(\lambda a^2, 2\lambda a x)|a|_p^{-(n+1)}\,dx\,da$$
$$= \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} \hat{\psi}(\cdot, \cdot, 0, 1)(\lambda a^2, x)|a|_p^{-(n+1)}\frac{|x|_p^n}{|2\lambda a|_p^n} = \frac{1}{|x|_p}\,dx\,da$$
$$= \int_{\mathbf{Q}_p^n \times \mathbf{Q}_p^s} \hat{\psi}(\cdot, \cdot, 0, 1)(\lambda a^2, x)\frac{1}{|\lambda|_p^n}|a|_p^{-(2n+1)}\,dx\,da.$$

We let $g \colon \mathbf{Q}_p \to \mathbf{C}$ be defined by $g(v) = \hat{\psi}(\cdot, \cdot, 0, 1)(\lambda v, x)|v|_p^{-(n+1)}$. Then $g \in C_c^\infty(\mathbf{Q})$ for each fixed x. Thus using Proposition 2.4

$$T_\lambda(\psi) = \int_{\mathbf{Q}_p^n} \int_{\mathbf{Q}_p^s} g(a^2)|a|_p \frac{1}{|\lambda|_p^n}\,da\,dx$$
$$= \frac{2}{|\lambda|_p^n} \int_{\mathbf{Q}_p^n} \int_{S(\mathbf{Q}_p^s)} \hat{\psi}(\cdot, \cdot, 0, 1)(\lambda a, x)|a|_p^{-(n+1)}\,da\,dx.$$

Now,

$$\psi\left(1_{N \cdot A}\right) = \psi\left(0, 0, 0, 1\right) = \int_{\mathbf{Q}_p^s} \int_{\mathbf{Q}_p^n} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(a, x\right) da\, dx$$

$$= \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(a, x\right) da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(ap, x\right) da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(au, x\right) da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(aup, x\right) da\, dx$$

$$= \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} D^{n+1} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(a, x\right) |a|_p^{-(n+1)} da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} D^{n+1} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(ap, x\right) |ap|_p^{-(n+1)} da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} D^{n+1} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(au, x\right) |au|_p^{-(n+1)} da\, dx$$

$$+ \int_{S(\mathbf{Q}_p^s)} \int_{\mathbf{Q}_p^n} D^{n+1} \psi\left(\cdot, \cdot, 0, 1\right)^{\widehat{}}\left(aup, x\right) |aup|_p^{-(n+1)} da\, dx$$

$$= \frac{1}{2} \operatorname{tr} \pi_1\left(D^{n+1} \psi\right) + \frac{p}{2} \operatorname{tr} \pi_p\left(D^{n+1} \psi\right)$$

$$+ \frac{1}{2} \operatorname{tr} \pi_n\left(D^{n+1} \psi\right) + \frac{p}{2} \operatorname{tr} \pi_{sp}\left(D^{n+1} \psi\right)$$

$$= T_1\left(\underline{D}\psi\right) + p T_p\left(\underline{D}\psi\right) + T_n\left(\underline{D}\psi\right) + p T_p\left(\underline{D}\psi\right).$$

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