TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 338, Number 2, August 1993

FIBERED PRODUCTS OF HOMOGENEOUS CONTINUA

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ABSTRACT. In this paper, we construct homogeneous continua by using a fibered product of a homogeneous continuum X with itself. The space X must have a continuous decomposition into continua, and it must possess a certain type of homogeneity property with respect to this decomposition. It is known that the points of any one-dimensional, homogeneous continuum can be "blown up" into pseudo-arcs to form a new continuum with a continuous decomposition into pseudo-arcs. We will show that these continua can be used in the above construction. Finally, we will show that the continuum constructed by using the pseudo-arcs, the circle of pseudo-arcs, or the solenoid of pseudo-arcs is not homeomorphic to any known homogeneous continuum.

INTRODUCTION

A continuum is a compact, connected metric space. A continuum X is homogeneous if, for any pair of points x, y in X, there exists a homeomorphism $h: (X, x) \to (X, y)$. In this paper, we will build new homogeneous continua by using fibered products of other homogeneous continua. If $f: X \to Z$ and $g: Y \to Z$ are maps, then the fibered product of X and Y with respect to f and g is the space $\{(x, y) \in X \times Y: f(x) = g(y)\}$.

One of the new homogeneous continua is the fibered product of two pseudo-arcs. A pseudo-arc is a chainable, hereditarily indecomposable continuum. A chainable continuum is a continuum which is homeomorphic to an inverse limit of arcs, and an indecomposable continuum is a continuum which is not the union of two of its proper subcontinua. A continuum is hereditarily indecomposable if every subcontinuum is indecomposable. Clearly, every nondegenerate subcontinuum of a pseudo-arc is a pseudo-arc. The pseudo-arc was first constructed by Knaster [7] in 1922. It was shown to be homogeneous by Bing [3] in 1948. Bing [2] also proved that all pseudo-arcs are homeomorphic.

A continuous decomposition of a continuum is a partition of the continuum into subcontinua such that the quotient map of the partition is both open and closed. A circle of pseudo-arcs is a continuum with a continuous decomposition into pseudo-arcs, such that the quotient space is a circle. Bing and Jones [4] constructed a circle of pseudo-arcs in 1954, and showed that any two circles of pseudo-arcs are homeomorphic, and that the circle of pseudo-arcs is homogeneous.

Received by the editors May 23, 1991.

1991 *Mathematics Subject Classification*. Primary 54F20.

Key words and phrases. Continuum, homogeneous, fibered product, aposyndetic continuum, continuous decomposition, pseudo-arc, circle of pseudo-arcs, solenoid of pseudo-arcs.

A solenoid is an inverse limit of circles with covering maps as the bonding maps. In 1977, for each solenoid S, Rogers [11] constructed a solenoid of pseudo-arcs, that is, a homogeneous continuum with a continuous decomposition into pseudo-arcs, such that the quotient space is S.

In 1984, Lewis [8] generalized the above results by showing that points of any homogeneous one-dimensional continuum X can be "blown up" into pseudo-arcs, so that the resulting continuum is a homogeneous continuum with a continuous decomposition into pseudo-arcs such that the quotient space is X.

In this paper, we will show that if Y is any of the continua constructed in Lewis' paper, a homogeneous continuum can be constructed as a fibered product of Y with itself. In particular, we will show that the continua constructed from the pseudo-arc, the circle of pseudo-arcs, and a solenoid of pseudo-arcs are not homeomorphic to any known homogeneous continua.

I

Let X be a continuum which has a continuous decomposition into continua, and let $f\colon X \to Q$ be the quotient map where Q is a homogeneous continuum. We will call the following property of X, with respect to f, Property H:

> If h is any homeomorphism of Q, and if $h(f(x)) = f(y)$, then there is a homeomorphism $\hat{h}\colon (X, x) \to (X, y)$ such that $f \circ \hat{h} = h \circ f$.

In this section, we will prove the following theorem.

Theorem 1. *Let X be a continuum with a continuous decomposition into nondegenerate continua, and let $f\colon X \to Q$ be the quotient map, where Q is a homogeneous continuum and X has Property* H *with respect to f. Let $\widetilde{X} = \{(x, y) \in X \times X\colon f(x) = f(y)\} = \bigcup\{f^{-1}(q) \times f^{-1}(q)\colon q \in Q\}$. Then $\widetilde{X}$ is an aposyndetic, homogeneous continuum such that for each $q \in Q$,*

$$\dim(f^{-1}(q) \times f^{-1}(q)) \leq \dim \widetilde{X} \leq \dim(X \times X).$$

Proof. Let $\pi_1\colon \widetilde{X} \to X$ and $\pi_2\colon \widetilde{X} \to X$ be projections onto the first and second coordinates, respectively. The map π_1 is open, because, if U_1 and U_2 are open subsets of X, then $\pi_1((U_1 \times U_2) \cap \widetilde{X}) = U_1 \cap f^{-1}(f(U_2))$ which is an open set since f is open. Also, note that for each $x \in X$, $\pi_1^{-1}(x) = \{x\} \times f^{-1}(f(x))$ is homeomorphic to a decomposition element of X.

The space $\widetilde{X}$ is compact, since it is a closed subset of $X \times X$. The space $\widetilde{X}$ is also connected. For, if $\widetilde{X} = U \cup V$, where U and V are nonempty, disjoint open sets, then $\pi_1(U)$ and $\pi_1(V)$ are nonempty open sets such that $\pi_1(U) \cup \pi_1(V) = X$. If $x \in \pi_1(U) \cap \pi_1(V)$, then $\{x\} \times f^{-1}(f(x))$ intersects both U and V. But $\{x\} \times f^{-1}(f(x))$ is connected, so it must be contained in one of these sets. Then $\widetilde{X}$ must be connected, and hence a continuum.

The space $\widetilde{X}$ is also homogeneous. If (x_1, y_1) and (x_2, y_2) are points of $\widetilde{X}$, let $p_1 = f(x_1) = f(y_1)$ and $p_2 = f(x_2) = f(y_2)$. Choose a homeomorphism $h\colon (Q, p_1) \to (Q, p_2)$. Then, by Property H, there are homeomorphisms $h_x\colon (X, x_1) \to (X, x_2)$ and $h_y\colon (X, y_1) \to (X, y_2)$, with $f \circ h_x = h \circ f = f \circ h_y$. We have $(h_x \times h_y)(\widetilde{X}) \subset \widetilde{X}$, for if $(x, y) \in \widetilde{X}$, then $f(x) = f(y)$, so

$$f(h_x(x)) = h(f(x)) = h(f(y)) = f(h_y(y)).$$

Also, if $(u, v) \in \widetilde{X}$, then $f(u) = f(v)$, so

$$\begin{aligned} h(f(h_x^{-1}(u))) &= f(h_x(h_x^{-1}(u))) = f(u) = f(v) \\ &= f(h_y(h_y^{-1}(v))) = h(f(h_y^{-1}(v))). \end{aligned}$$

Since h is a homeomorphism, $f(h_x^{-1}(u)) = f(h_y^{-1}(v))$, and $(h_x^{-1}(u), h_y^{-1}(v)) \in \widetilde{X}$. Then $(h_x \times h_y)(\widetilde{X}) = \widetilde{X}$, and therefore, $h_x \times h_y$ is a homeomorphism of $\widetilde{X}$ that takes (x_1, y_1) to (x_2, y_2).

A continuum Y is aposyndetic if for each pair of distinct points x and y in Y, there is a subcontinuum S of Y such that $x \in \operatorname{int}(S)$ and $y \in Y - S$. Jones [6] was the first to use this concept to study homogeneous continua.

We will next show that $\widetilde{X}$ must be aposyndetic. Let (x, y) and (x', y') be distinct points in $\widetilde{X}$. Without loss of generality, assume $x \neq x'$. Let U be an open subset of X such that $x \in U$ and $x' \notin \overline{U}$. Then $\pi_1^{-1}(\overline{U})$ is a closed set containing (x, y) in its interior, but not containing (x', y'). Since $\pi_1^{-1}(\overline{U}) = \bigcup\{\{z\} \times f^{-1}(f(z)) \colon z \in \overline{U}\}$, it is a union of connected sets. Let $\hat{h}$ be a homeomorphism of X which fixes the decomposition elements, such that $\hat{h}(x') \neq y'$. We can obtain $\hat{h}$ by using Property H, and letting the function h in the definition of this property be the identity homeomorphism. Then the set $A = \{(z, \hat{h}(z)) \colon z \in X\} \subseteq \widetilde{X}$ is a continuum, since it is homeomorphic to X. Also, $(x', y') \notin A$, and for every $z \in \overline{U}$,

$$A \cap [\{z\} \times f^{-1}(f(z))] \neq \varnothing.$$

Then $\pi_1^{-1}(\overline{U}) \cup A$ is a continuum containing (x, y) in its interior, but not containing (x', y').

We also know that for each $q \in Q$, we have

$$\dim(f^{-1}(q) \times f^{-1}(q)) \leq \dim(\widetilde{X}) \leq \dim(X \times X),$$

since $f^{-1}(q) \times f^{-1}(q) \subset \widetilde{X} \subset X \times X$. $\square$

We can generalize the above result. If $\mathscr{A}$ is any indexing set, and X is a continuum with a continuous decomposition into nondegenerate subcontinua, with quotient space the homogeneous space Q, and having Property H with respect to the quotient map f, then the space $\bigcup_{q \in Q} \prod_{\alpha \in \mathscr{A}} f^{-1}(q)$ is an aposyndetic, homogeneous continuum. The proof is analogous to the proof of Theorem 1. Also, suppose X and Y are two continua with continuous decompositions into nondegenerate subcontinua, both with quotient space the homogeneous space Q. Suppose X has Property H with respect to the quotient map f, and Y has Property H with respect to the quotient map g. Then a proof analogous to the proof of Theorem 1 shows that $\bigcup_{q \in Q}(f^{-1}(q) \times g^{-1}(q))$ is a homogeneous continuum. However, the author is not able to conclude that this space is aposyndetic.

II

Lewis [8] has shown that if Q is a one-dimensional, homogeneous continuum, then there is a one-dimensional continuum X with a continuous decomposition into pseudo-arcs with quotient map $f \colon X \to Q$, such that X satisfies Property H with respect to f. Then $\widetilde{X}$, as constructed in §I, would be a two-dimensional, homogeneous, aposyndetic continuum. Since the pseudo-arc is acyclic, the Vietoris-Begle Theorem tells us that the Čech cohomology of $\widetilde{X}$ is the same as the Čech cohomology of Q.

Three continua with which we could replace Q in the above paragraph are the circle, a solenoid, and the pseudo-arc. The corresponding continua with continuous decompositions into pseudo-arcs are the circle of pseudo-arcs (CP), a solenoid of pseudo-arcs (SP), and the pseudo-arc of pseudo-arcs (P), which is known to be homeomorphic to the pseudo-arc [9]. We will show that $\widetilde{CP}$, $\widetilde{SP}$, and $\widetilde{P}$ are not homeomorphic to any previously known homogeneous continuum. J. T. Rogers has informed the author that, in the case of $\widetilde{CP}$, this solves a problem posed by the late Andrew Conner about 10 years ago.

Rogers [10] has classified homogeneous continua as being locally connected, aposyndetic but not locally connected, decomposable but not aposyndetic, or indecomposable (with further subcategories for indecomposable continua.) He lists all the known homogeneous continua in each category. $\widetilde{CP}$, $\widetilde{SP}$, and $\widetilde{P}$ are two-dimensional continua and belong to the category aposyndetic, but not locally connected. They are not locally connected because their images under π_1 are not locally connected.

The known two-dimensional homogeneous continua in this category are products of two one-dimensional homogeneous continua, at least one of which is not locally connected, and a few other continua which contain arcs. None of $\widetilde{CP}$, $\widetilde{SP}$, and $\widetilde{P}$ can contain an arc, because the image of an arc under π_1 would have to be locally connected. The only locally connected subcontinua of CP, SP, or P are points. If the image of an arc under π_1 was a point, the arc would have to be contained in a pseudo-arc, which contains no arcs.

The known one-dimensional homogeneous continua are S^1, the Menger curve, the Case continua, other Cantor set bundles over the Menger curve, the solenoids, the pseudo-arc, the circle of pseudo-arcs, the Menger curve of pseudo-arcs, the Case continua of pseudo-arcs, the Cantor set bundles over the Menger curve of pseudo-arcs, and the solenoids of pseudo-arcs [10]. It is easy enough to list all of the products of two one-dimensional homogeneous continua, at least one of which is not locally connected, and note which have the cohomology of $\widetilde{CP}$, $\widetilde{SP}$, or $\widetilde{P}$. The continua in the list with the cohomology of $\widetilde{CP}$ are $S^1 \times P$ and $CP \times P$. The continua with the cohomology of $\widetilde{SP}$ are $S \times P$ and $SP \times P$. The only continuum in the list with the cohomology of $\widetilde{P}$ is $P \times P$. Since $S^1 \times P$ and $S \times P$ contain arcs, the only homogeneous continua to which $\widetilde{CP}$, $\widetilde{SP}$, and $\widetilde{P}$ could be homeomorphic are $CP \times P$, $SP \times P$, and $P \times P$, respectively.

III

In this section, we will show that $\widetilde{SP}$ is not homeomorphic to $SP \times P$, and $\widetilde{P}$ is not homeomorphic to $P \times P$. A fact that we will need in this section and the next is that the decompositions of CP, SP, and P are terminal. This means that each subcontinuum is either contained in a decomposition element, or is a union of decomposition elements. Hence, the only proper, nondegenerate subcontinua of CP and SP are arcs of pseudo-arcs and pseudo-arcs.

Now we will prove a couple of lemmas.

Lemma 1. *Let X be CP, SP, or P, and let K be a subcontinuum of $\widetilde{X}$ such that $\pi_1(K)$ intersects more than one decomposition element. Then $\pi_2(K) = \pi_1(K)$.*

Proof. Let D be a decomposition element which intersects $\pi_1(K)$, and let $(x, y) \in K \cap \pi_1^{-1}(D)$. Then we must have $y \in D$; so D intersects $\pi_2(K)$. Likewise, any decomposition element which intersects $\pi_2(K)$ also intersects $\pi_1(K)$. Since $\pi_1(K)$ and $\pi_2(K)$ are subcontinua of X which intersect more than one decomposition element, and since the decomposition is terminal, $\pi_1(K)$ and $\pi_2(K)$ must each be a union of the decomposition elements it intersects, so $\pi_1(K) = \pi_2(K)$. □

A map $g: Y \to Z$ between continua is weakly confluent if, for each subcontinuum S of Z, there is a component of $g^{-1}(S)$ which maps onto S. Any map of a continuum onto a pseudo-arc, an arc of pseudo-arcs, or a solenoid of pseudo-arcs is weakly confluent [5].

Recall that pseudo-arcs are chainable. We will use the following fact concerning chainable continua in the next lemma: If C_1 and C_2 are subcontinua of $D \times D$, where D is a chainable continuum, and the projection of C_1 onto the first coordinate is D, and the projection of C_2 onto the second coordinate is D, then $C_1 \cap C_2 \neq \varnothing$ [1].

Lemma 2. *If K_1 and K_2 are subcontinua of $\widetilde{SP}$ or $\widetilde{P}$ such that $\pi_1(K_1)$ and $\pi_1(K_2)$ are each unions of more than one decomposition element, and $\pi_1(K_1) \cap \pi_1(K_2) \neq \varnothing$, then $K_1 \cap K_2 \neq \varnothing$.*

Proof. Since $\pi_1(K_1)$ and $\pi_1(K_2)$ must each be either a pseudo-arc, an arc of pseudo-arcs, or SP, and since $\pi_2(K_2) = \pi_1(K_2)$, we know that the maps $\pi_1|K_1$ and $\pi_2|K_2$ are both weakly confluent. There is a decomposition element $D \subset \pi_1(K_1) \cap \pi_1(K_2) = \pi_1(K_1) \cap \pi_2(K_2)$ since $\pi_1(K_1) \cap \pi_1(K_2) \neq \varnothing$, and $\pi_1(K_1)$ and $\pi_1(K_2)$ are each unions of decomposition elements. Then there exists subcontinua C_1 of K_1 and C_2 of K_2 such that $\pi_1(C_1) = D$ and $\pi_2(C_2) = D$. Then $\pi_2(C_1) \subset D$ and $\pi_1(C_2) \subset D$, since the coordinates of points of $\widetilde{P}$ and $\widetilde{SP}$ must come from the same decomposition element. Then C_1 and C_2 are subcontinua of $D \times D$ such that $\pi_1(C_1) = D$ and $\pi_2(C_2) = D$. Since D is chainable, we must have $C_1 \cap C_2 \neq \varnothing$. Then $K_1 \cap K_2 \neq \varnothing$. □

Let X be SP or P. Let $p_1: X \times P \to X$ and $p_2: X \times P \to P$ be the projections. If $x \in X$, we will call $p_1^{-1}(x)$ a vertical slice of $X \times P$, and if $q \in P$, we will call $p_2^{-1}(q)$ a horizontal slice of $X \times P$.

Theorem 2. *$\widetilde{SP}$ is not homeomorphic to $SP \times P$, and $\widetilde{P}$ is not homeomorphic to $P \times P$.*

Proof. Let X be SP or P, and assume $\sigma: X \times P \to \widetilde{X}$ is a homeomorphism.

Step 1. We will show that if h_1 and h_2 are distinct horizontal slices of $X \times P$, and both $\pi_1(\sigma(h_1))$ and $\pi_1(\sigma(h_2))$ are unions of more than one decomposition element, then $\pi_1(\sigma(h_1))$ and $\pi_1(\sigma(h_2))$ do not intersect.

If $\pi_1(\sigma(h_1))$ and $\pi_1(\sigma(h_2))$ intersect, then by Lemma 2, $\sigma(h_1)$ and $\sigma(h_2)$ intersect. But h_1 and h_2 are distinct horizontal slices, and σ is a homeomorphism, so $(\sigma(h_1))$ and $(\sigma(h_2))$ cannot intersect.

Step 2. We will show that there is at least one horizontal slice h of $X \times P$ such that $\pi_1(\sigma(h))$ is contained in a decompostion element.

If for every horizontal slice h of $X \times P$, $\pi_1(\sigma(h))$ is a union of more than one decomposition element, then Step 1 tells us that for each horizontal slice h, $\sigma(h) = \pi_1^{-1}(\pi_1(\sigma(h)))$. But $\sigma(h)$ must be one-dimensional, while $\pi_1^{-1}(\pi_1(\sigma(h)))$ is two-dimensional, since $\pi_1(\sigma(h))$ is a union of more than one decomposition element. Hence, there is a horizontal slice h such that $\pi_1(\sigma(h))$ is contained in a decomposition element.

Step 3. Now we prove the theorem. Let h be a horizontal slice of $X \times P$ such that $\pi_1(\sigma(h))$ is contained in a decomposition element D. Let v_1 be a vertical slice of $X \times P$ such that $\pi_1(\sigma(v_1))$ intersects $X \backslash D$. Since $\pi_1^{-1}(X \backslash D)$ is two-dimensional, $\pi_1^{-1}(X \backslash D) \not\subset \sigma(v_1)$, so there exists a vertical slice v_2 of $X \times P$ distinct from v_1 such that $\pi_1(\sigma(v_2))$ intersects $X \backslash D$. Both $\sigma(v_1)$ and $\sigma(v_2)$ must intersect $\sigma(h)$, since both v_1 and v_2 intersect h, so $\pi_1(\sigma(v_1))$ and $\pi_1(\sigma(v_2))$ each also intersect D. Since $\pi_1(\sigma(v_1))$ and $\pi_1(\sigma(v_2))$ each also intersect $X \backslash D$, they must each be unions of more than one decomposition element, and must each contain D. Then by Lemma 2, $\sigma(v_1) \cap \sigma(v_2) \neq \varnothing$. But this is a contradiction since $v_1 \cap v_2 = \varnothing$. Hence, there can be no homeomorphism $\sigma: X \times P \to \widetilde{X}$.

IV

In this section, we will prove that $CP \times P$ is not homeomorphic to $\widetilde{CP}$. We will use a lemma similar to Lemma 2. Since CP does not satisfy the property that every map of a continuum onto it is weakly confluent, the hypothesis for this lemma is more restrictive than the hypothesis of Lemma 2.

Lemma 3. *If K_1 and K_2 are subcontinua of $\widetilde{CP}$ such that $\pi_1(K_1)$ and $\pi_1(K_2)$ are arcs of pseudo-arcs, and $\pi_1(K_1) \cap \pi_1(K_2) \neq \varnothing$, then $K_1 \cap K_2 \neq \varnothing$.*

Proof. The proof is analogous to the proof of Lemma 2. □

If $p_1: CP \times P \to CP$ and $p_2: CP \times P \to P$ are the projections, then for each $x \in CP$, we will call $p_1^{-1}(x)$ a vertical slice of $CP \times P$, and for each $q \in P$, we will call $p_2^{-1}(q)$ a horizontal slice of $CP \times P$.

Theorem 3. *$CP \times P$ is not homeomorphic to $\widetilde{CP}$.*

Proof. Assume $\sigma: CP \times P \to \widetilde{CP}$ is a homeomorphism.

Step 1. We will show that if k is a horizontal or vertical slice of $CP \times P$, then $\pi_1(\sigma(k))$ is contained in a decomposition element.

Suppose $\pi_1(\sigma(k))$ is not contained in a decomposition element. Then $\pi_1(\sigma(k))$ contains an arc of pseudoarcs A. Choose a sequence k_n of vertical slices, if k is vertical, or horizontal slices, if k is horizontal, of $CP \times P$, such that $k_n \neq k$ for all n, and $k_n \to k$. Since $f(\pi_1(\sigma(k_n))) \to f(\pi_1(\sigma(k)))$, we must have, for n large, $f(\pi_1(\sigma(k_n)))$ is nondegenerate. Also, since $\operatorname{int}(A)$ is an open set containing points of the limit set of $\{\pi_1(\sigma(k_n))\}$, for n large, $\pi_1(\sigma(k_n))$ must intersect $\operatorname{int}(A)$. Choose n so that $\pi_1(\sigma(k_n))$ contains an arc of pseudoarcs intersecting $\operatorname{int}(A)$, and choose a decompostion element $D \subset \operatorname{int}(A) \cap \pi_1(\sigma(k_n))$. Let S be a component of $\pi_1^{-1}(A) \cap \sigma(k)$, and S_n be a component of $\pi_1^{-1}(A) \cap \sigma(k_n)$, such that $\pi_1(S)$ and $\pi_1(S_n)$ each intersect D. We will show that $\pi_1(S)$ and $\pi_1(S_n)$ must each be arcs of pseudo-arcs containing D.

If $\pi_1(S)$ is not an arc of pseudo-arcs, then $\pi_1(S) \subset D$, and hence $S \neq \sigma(k)$. If Y is a subcontinuum of $\sigma(k)$ properly containing S, then $\pi_1(Y)$ must intersect the complement of A, since S is a component of $\pi_1^{-1}(A) \cap \sigma(k)$. Since S is a proper subcontinuum of $\sigma(k)$, a space homeomorphic to either a pseudo-arc or a circle of pseudo-arcs, we can find a decreasing sequence $\{Y_n\}$ of subcontinua of $\sigma(k)$, each properly containing S, such that $Y_n \to S$. Then $\pi_1(Y_n) \to \pi_1(S)$. However, for each n, $\pi_1(Y_n)$ intersects the complement of A, while $\pi_1(S) \subset D \subset \operatorname{int}(A)$. Since this is impossible, $\pi_1(S)$ must be an arc of pseudo-arcs. Likewise, we can show that $\pi_1(S_n)$ is an arc of pseudo-arcs.

Since both $\pi_1(S)$ and $\pi_1(S_n)$ are arcs of pseudo-arcs containing D, by Lemma 3, $S \cap S_n \neq \varnothing$, and $\sigma(k) \cap \sigma(k_n) \neq \varnothing$. But k and k_n are distinct, so this is impossible. Therefore, $\sigma(k)$ must be contained in a decomposition element.

Step 2. Now we prove the theorem. Let h be a horizontal slice of $CP \times P$. Then $\pi_1(\sigma(h))$ is contained in a decomposition element D. If v is any vertical slice of $CP \times P$, $\pi_1(\sigma(v))$ must intersect D since v intersects h. Since $\pi_1(\sigma(v))$ is contained in a decomposition element, $\pi_1(\sigma(v)) \subset D$. Since $CP \times P$ is a union of vertical slices, we have $\sigma(CP \times P) \subset \pi_1^{-1}(D)$, so σ could not be a homeomorphism.

Thus, $\widetilde{CP}$, $\widetilde{SP}$, and $\widetilde{P}$ are new examples of homogeneous continua.

Acknowledgment

It is acknowledged that this research was a dissertation done at Tulane University under the direction of Professor James T. Rogers, Jr.

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